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# IDENTIFICATION OF NON-LINEAR DYNAMIC SYSTEM

## *Digest of paper*[1]

**Ventseslav Shopov and Vanya Markova**

*Institute of Robotics Bulgarian Academy of Sciences,*
*139 Rousski blvd., 4000 Plovidv*
*e-mails: {vkshopov, narkovavanya}@yahoo.com*
*Bulgaria*

**Abstract:** The behaviour of non-linear dynamic systems is studied. Through numerical experiments the coefficients of the non-linear differential equations are identified. In this paper, the authors present an investigation of the modelling and prediction abilities of a traditional Recurrent Neural Network (RNN) and a Long Short-Term Memory (LSTM) RNN, when the input signal has a chaotic nature. The effectiveness of both networks in modelling the Lorenz System is studied. And a comparison of their respective one-step ahead predictions is made.
**Key words:** Recurrent Neural Network, Long Short-Term Memory component, Lorenz System.

## 1. INTRODUCTION

One of the central questions in science is forecasting: given the past, what is the optimal future prediction? In many domains with complex spatio-temporal structure and non-linear dynamics, such as weather systems and transportation networks, forecasting poses a significant unsolved structured prediction challenge. The key issue is how to accurately capture the non-linear dynamics and higher order correlations of the data-generating process, which can make systems highly sensitive to initial conditions [1].

Common approaches to forecasting involve using spectral analysis, discrete time models, functional interpolation and explicitly learning the system dynamics (system identification) [4]. Other methods have used hierarchical models that learn both short-term and long-term behaviour of the dynamics [5] from data. However, it is often infeasible to obtain good analytical dynamics models or stable predictions.

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[1] The full paper is proposed for including in the IEEE Xplore Digital Library

This work is related to classic work in time series fore- casting, which has studied auto-regressive models, such as the ARMA or ARIMA model [6]. These model a process x(t) linearly, which does not capture non-linear, chaotic systems. Using neural networks to capture the chaotic dynamics has a long history [7] and have been applied to weather forecasting, traffic prediction and other domains [8]. From a modelling perspective, considers a high-order RNN to simulate a deterministic finite state machine and recognize regular grammars.

We study the effectiveness of neural networks in modelling the Lorenz, Roessler and Burke-Shaw systems and compare their predictions. The paper is organized as follows: in Section 2, we describe the chaotic systems and network models used; Section 3 presents the simulation results, as well as the results of experiments. Finally, section 4 presents the conclusions and possible future work.

## 2. METHODS AND MATERIALS

### 2.1. Chaotic attractors

In [10] a non-linear autonomous system of differential equations (dynamical system) was defined as follow:

*Table 1.Chaotic attractors*

| | | |
|---|---|---|
| $\frac{dx}{dt} = \sigma(y - x)$<br>$\frac{dy}{dt} = x(\rho - z) - y$<br>$\frac{dz}{dt} = xy - \beta z$ | $\frac{dx}{dt} = -s(x + y)$<br>$\frac{dy}{dt} = -y - s\,x\,z$<br>$\frac{dz}{dt} = s\,x\,y + v$ | $dx/dt = -(y + z)$<br>$dy/dt = x + a * y$<br>$dz/dt = b + z * (x - c)$ |
| Lorenz attractor | Roessler attractor | Burke-Shaw attractor |

Numerical methods are often used to analyse the structure of the attractor. In the case of classical values of the system parameters, instability of its solutions is observed. This is because the equilibrium positions of the system are of saddle type. As a sequence this limits the use of these methods, since the total error increases with an increase in the integration interval. So small changes in the initial conditions of the system can lead to significant consequences over time.

### 2.2. Recurrent Neural Networks

Basic RNNs are a network of neuron-like nodes organized into successive "layers", each node in a given layer is connected with a directed (one-way) connection to every other node in the next successive layer. Each node (neuron) has a time-varying real-valued activation.

There are several architectures of LSTM units. A common architecture is composed of a memory cell, an input gate, an output gate and a forget gate. Gated Recurrent Unit (GRU) are improvement of LSTM network as is described in [11].

### 2.3. Implementation

In this study we examine the convergence of the loss function as an assessment of the convergence of the learning process. In addition, we compare the performance of three recurrent methods on three chaotic attractors.

## 3. EXPERIMENTS AND RESULTS

All the traditional RNN, LSTM and GRU were applied to chaotic data generated from chaotic systems. Results for the network training and evaluation of the networks predictive capabilities on the data are described below. We have train all networks on imput vectros with variing size from 4000 to 14000 samples and validating on last 1000 samples. We build the datasets as follows: we solve differential equations and as a result, build time series with a length of 15000 elements. Then build training sets with a length of 4,000 to 14,000 items. Then we build a validation set with a length of 1000 elements.

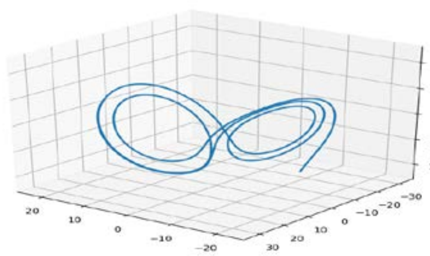

*Fig. 1. Actual Lorenz attractor*

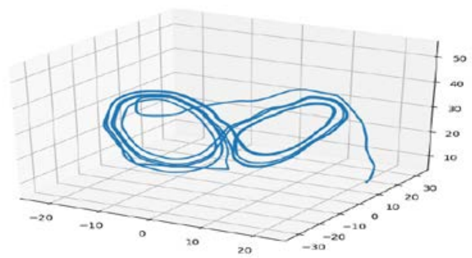

*Fig. 2. Predicted Lorenz attractor using RNN*

In the second step, we assign the hyper-parameters of our recurrent neuron networks. We use a variable length input layer. The output layer is a three-element vector [x, y, z]. We choose the hyperbolic tangent as an activation function. For kernel initialization, we use uniform Glorot initialization. We apply ADAM as optimizer and MSE as a loss function. To evaluate the quality of the built model, we use the MSE as a metric again.

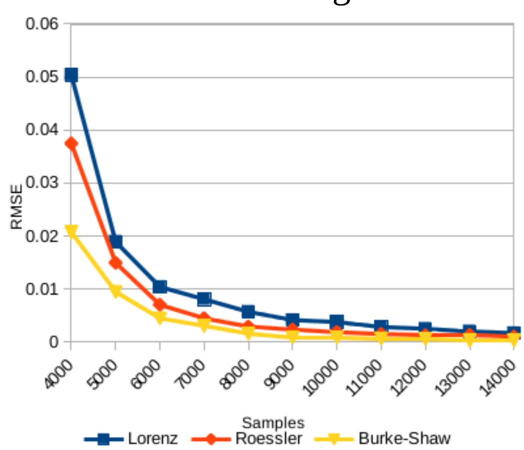


*Fig. 3. Comparison of the complexity of predicting three different RNN attractors according to the length of the time series.*

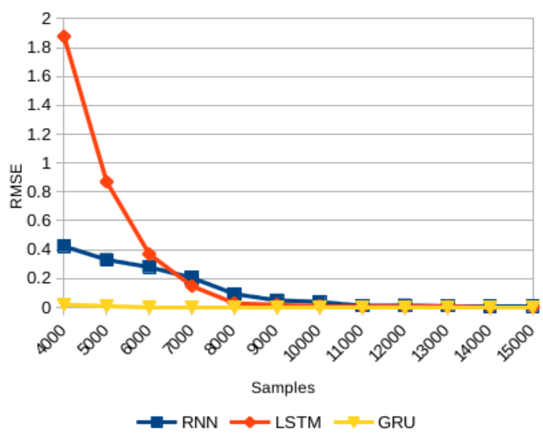


*Fig. 4. Comparison of the Burke-Shaw Attractive Prediction Prediction with three different Recurrent Networks according to the*

*length of the time series.*

From the second experiment, it can be seen that for time series with less than 6,000 elements, the LSTM is not particularly effective. Even the simplest RNN gets better results up to series with lengths of 7,000 samples. However, when we have more than 7,000 elements, the situation changes and the LSTM shows better results than RNN. Nevertheless, the GRU demonstrates significantly better results than both RNN and LSTM. Again, it can be seen that for time series with lengths of more than 10000 elements the differences in accuracy become less than 0.01 for all three recurrent neural methods.

## 4. CONCLUSION

In this article, deep neural networks RNN and LSTM are used to model a chaotic system and to predict the status of the system one step ahead. Experiments show that the LSTM network performs better with the traditional RNN. This is due to the increased LSTM capabilities for studying long-term dependencies.

The chaotic systems survey can provide useful information about weather forecasts, traffic forecasts, and stock market forecasts. As far as future work is concerned, it would be interesting to study other types of NNs and to investigate their effectiveness in a prediction of chaotic systems compared to RNN and LSTM.

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