Proceedings of the
35th International Conference on Information Technologies (InfoTech-2021)
IEEE Conference, Rec. # 52438, 16-17 September 2021, Bulgaria

A SURVEY OF PERIODIC BINARY NEARLY PERFECT SIGNALS WITH LENGTHS N≡0MOD4

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Abstract: In the paper the results of our survey of periodic binary nearly perfect signals with lengths N≡0mod4 are presented. These signals possess autocorrelation functions, which have both the possibly smallest quantity and magnitude of side lobes and, due to this reason, they are named nearly perfect. A very important positive feature of the periodic binary nearly perfect signals is the possibility to be processed by the so-called side lobes suppression filters with very small losses in the signal-to-noise ratio. The results, obtained in the paper, could be useful in development of different types of radar sensors and radio-synchronization devices.
Key words: digital signal processing, periodic binary nearly perfect signals.

1. INTRODUCTION

Today the IV-th industrial revolution is in a rapid progress. Many technologies and specialized systems play significant role for its success. For example, the complex sensor networks, built of small radar and/or sonar sensors, measuring precisely the distances among great number of moving objects, are compulsory component of the present manufacture, smart cars, smart homes and so on [1, 2].

The performance effectiveness of the small radar sensors depends essentially on the correlation properties of the exploited systems of radio signals, as well on the complexity of their generation and processing. Due to this reason today the binary phase manipulated (PM) signals, which are the simplest from implementation point of view, are object of special interest. In this research area the main objective are the methods for synthesis of binary PM signals with ideal autocorrelation functions (ACFs), resembling the Dirac delta – pulse, because they provide the maximal possible resolution of the objects and diminish the negative effects, caused by the multipath spread of the electromagnetic waves and by the simultaneous asynchronous work of multiple sensors [3-7]. As it is very possible that the signals with ideal ACFs form a class, consisting of only one member [4-6], the research interest is focused on the periodic binary nearly perfect PM signals with lengths N≡0mod4. In the sequel for brevity the curtailment “BNPS” will be used instead of “binary nearly perfect PM signals”. Besides, the characteristic “PM” will be retained for signals, which phase modulation can be more complicate than the binary phase shift keying (BPSK).

The positive features of BNPSs can be summarized briefly as follows. First, BNPSs possess periodic ACFs (PACFs), which have both the possibly smallest quantity and magnitude of side lobes for every specific length N. Second, the BNPSs can be processed by the so-called side lobes suppression filters (SLSFs) with very small losses in the signal-to-noise ratio (SNR) [3, 6, 8-12]. As a result, the BNPSs can be as beneficial for the radio-communication systems (RCSs) as the signals with ideal PACFs from the point of view of signal resolution.

Accounting the above facts, in the paper the results of our survey of BNPSs with lengths N≡0mod4 are presented. They expand the results, obtained in our previous paper [9].

The paper is organized as follows. First, the basic mathematical model of the BNPSs is recalled. After that several restrictions, diminishing the computational load during the survey, are proposed. On this base the methodology of the survey, developed in our previous papers [8-11], is elaborated. At the end the effectiveness of the modified methodology is demonstrated by several examples of unknown up to now BNPSs with lengths N≡0mod4, synthesized by the computer program, developed on its base.

2. METHODOLOGY OF CONDUCTED SURVEY OF PERIODIC BINARY NEARLY PERFECT SIGNALS WITH LENGTHS N≡0MOD4

The signal families with good (or optimal) correlation properties have great importance for the RCSs in general, because the signal processing in RCS receivers begins with evaluation of ACFs or PACFs of the detected signals as this procedure maximizes the SNR [6]. As a result, the methods for synthesis of signal families with good (or optimal) correlation properties have been intensively explored during the past seventy years. Despite of all taken efforts, at the moment only few classes of perfect signals are discovered [4-7]. This situation can be explained by the fact that this problem may be extremely hard from theoretical point of view. In fact, the linear increasing of the signal length N leads to exponential increasing of the computer load during the synthesis of these signal families, which makes impossible the finding of solutions for an acceptable long period of time even though by very powerful computer systems. Taking into consideration this situation the universal computer program, developed for our survey, uses several restrictions, which exclude all repeating information or simplify the computational procedures. In order to substantiate these restrictions firstly the model of digital processing of the PM signals will be recalled.

As known [3-6, 8-12], every periodic nearly perfect PM signal has the following PACF:

$$q_{\varsigma\varsigma}(r)=\sum_{i=0}^{N-1}\varsigma(i)\varsigma^{*}\langle i+r\rangle=\begin{cases}N, r=0,\\ q_{\min}, r\neq 0.\end{cases} \tag{1}$$

In (1) $q_{\varsigma\varsigma}(r)$ is the sample of the discrete PACF of the periodic nearly perfect PM signal at discrete time-shift r, the symbol "$\langle i+r\rangle$" means "summation modulo N", the symbol "*" stands for "complex conjugation". Besides, in (1) it is used the fact that every periodic PM signal with period $T_{rep}=N\tau_{ch}$ can be presented as a digital signal (sequence):

$$\{\varsigma(i)\}_{i=0}^{N-1}=\{\varsigma(0),\varsigma(1),...,\varsigma(N-1)\}, \tag{2}$$

consisting of the complex numbers $\varsigma(0),\varsigma(1),...,\varsigma(N-1)$, which are the complex envelopes of the elementary phase chips with duration τ_{ch}, forming the PM signal. In the class of the binary signals, which are the simplest PM signals from implementation point of view, all the samples $\varsigma(i)$ are -1 or $+1$. In this situation it is proved [4] that

$$q_{\min}\equiv N \bmod 4. \tag{3}$$

The results, obtained up to now [4] - [7], show that the PACFs of the periodic binary perfect and BNPSs have non-zero side-lobes with only one exception (the so-called Barker signal with length $N=4$ [4-7]):

$$\{\varsigma(i)\}_{i=0}^{3}=\{1,1,-1,1)\}. \tag{4}$$

As a result, frequently the side-lobes of the PACFs of the more powerful signals mask the main-lobes of the PACFs of the weaker signals. These situations may be very undesirable, because the weaker signals could be more important for the RCS. Here should be taken into consideration the fact that in the RCS receivers PM signals can be processed by the so-called side-lobe suppression filters (SLSF), which eliminate all the side-lobes of the PACF [3, 6, 8-12]. From the point of view of signal resolution, the replacement of classic matched filter (MF) by respective SLSF in the receivers can make some PM signals as beneficial for RCSs as the unique Barker signal (4). These arguments demonstrate that the exploration of the BNPSs, which can be processed by SLSFs with small losses in the SNR, have a great practical importance. Accounting this conclusion, the universal computer program for search of such signals, presented in our previous paper [9], has been modified and elaborated. The restrictions, used in it, can be described briefly as follows.

R 1) *Focusing the search on the BNPSs with length* $N \equiv 0 \bmod 4$.

The application of this restriction can be explained by the fact that according to (3), the BNPSs with length $N \equiv 0 \bmod 4$ may have PACFs with large zones with zero-level side-lobes. As a result, the negative effect of masking the main-lobes of the PACFs of the weaker signals by the side-lobes of the PACFs of the more powerful signals can occur very seldom. This fact will be explained in more detail in the next section of the paper, where the main results of the conducted survey of the BNPSs with length $N \equiv 0 \bmod 4$ are presented.

R 2) *Precise determination of the quantities of samples* -1 *and* $+1$ *of the synthesized BNPSs.*

This restriction ensues from the following analysis. Let

$$F_{\varsigma}(x) = \varsigma(0) + \varsigma(1)x + ... + \varsigma(N-1)x^{N-1} = \sum_{i=0}^{N-1} \varsigma(i)x^{i} \qquad (5)$$

be the so-called Hall polynomial, associated with the PM signal (2) [4, 8-12].

Analogously, let $F_{\varsigma}^{*}(x^{-1})$ and $Q_{\varsigma\varsigma}(x)$ be the Hall polynomials, associated with the MF of the PM signal (2) and to its PACF respectively:

$$F_{\varsigma}^{*}(x^{-1}) = \sum_{i=0}^{N-1} \varsigma^{*}(i)x^{-i}\,, \qquad (6)$$

$$Q_{\varsigma\varsigma}(x) = \sum_{i=0}^{N-1} q_{\varsigma\varsigma}(i)x^{i} = N + \sum_{i=1}^{N-1} q_{\varsigma\varsigma}(i)x^{i}\,. \qquad (7)$$

In (5) and (6) $\forall\varsigma^{*}(i) = \varsigma(i) \in \{-1,1\}$. Besides, in (7) $q_{\varsigma\varsigma}(0) = N$ is the main-lobe and

$$q_{\varsigma\varsigma}(1), q_{\varsigma\varsigma}(2), ..., q_{\varsigma\varsigma}(N-1) \equiv 0 \bmod 4\,, \qquad (8)$$

$$q_{\varsigma\varsigma}(i) = q_{\varsigma\varsigma}(N-i),\ i = 1, 2, ..., N/2\,, \qquad (9)$$

are the side lobes of the PACF [4-7].

Here it should be recalled that PACFs of BNPSs have both the possibly smallest quantity and magnitude of side lobes for every specific length N. From this fact and (3) can be concluded that the side lobes (8) belong to the set

$$\forall q_{\varsigma\varsigma}(i) \in \{-4, 0, +4\}\,. \qquad (10)$$

The polynomials (5), (6) and (7) allow the PACF evaluation by its MF to be presented in the form

$$F_{\varsigma\varsigma}(x)F_{\varsigma\varsigma}^{*}(x^{-1}) \equiv Q_{\varsigma\varsigma}(x) \bmod (x^{N} - 1)\,. \qquad (11)$$

The polynomial congruency (9) is equivalent to the following system of equations

$$\left|\begin{array}{c} F_{\varsigma\varsigma}(w^0)F^*_{\varsigma\varsigma}(w^0) = Q_{\varsigma\varsigma}(w^0) \\ F_{\varsigma\varsigma}(w^1)F^*_{\varsigma\varsigma}(w^{-1}) = Q_{\varsigma\varsigma}(w^1) \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ F_{\varsigma\varsigma}(w^{N-1})F^*_{\varsigma\varsigma}(w^{-(N-1)}) = Q_{\varsigma\varsigma}(w^{N-1}) \end{array}\right. \quad (12)$$

Here

$$w^k = e^{j\frac{2\pi}{N}k}, k = 0,1,\dots,N-1 \quad (13)$$

are all the zeros of the moduli polynomial $x^N - 1$ in the field of the complex numbers C.

As $N \equiv 0 \bmod 4$, the set

$$w^0 = e^{j\frac{2\pi}{N}0} = 1, w^{\frac{N}{2}} = e^{j\frac{2\pi}{N}\frac{N}{2}} = -1, w^{\frac{N}{4}} = e^{j\frac{2\pi}{N}\frac{N}{4}} = j = \sqrt{-1} \quad (14)$$

is a subset of the set (13) for every specific signal length N, analysed in the sequel. This fact allows the system of equations (12) to be reduced to the form:

$$\left|\begin{array}{l} [S(0)+S(1)+S(2)+S(3)]^2 = Q(0)+Q(1)+Q(2)+Q(3) \\ [S(0)-S(1)+S(2)-S(3)]^2 = Q(0)-Q(1)+Q(2)-Q(3) \\ \{[S(0)-S(2)]+j[S(1)-S(3)]\}\{[S(0)-S(2)]-j[S(1)-S(3)]\} = \\ \qquad = [Q(0)-Q(2)]+j[Q(1)-Q(3)] \end{array}\right. \quad (15)$$

In (15) $S(0),S(1),S(2),S(3)$ and $Q(0),Q(1),Q(2),Q(3)$ stand for:

$$\begin{array}{c} S(0) = \varsigma(0)+\varsigma(4)+\dots+\varsigma(N-4),\ S(1) = \varsigma(1)+\varsigma(5)+\dots+\varsigma(N-3), \\ S(2) = \varsigma(2)+\varsigma(6)+\dots+\varsigma(N-2),\ S(3) = \varsigma(3)+\varsigma(7)+\dots+\varsigma(N-1), \end{array} \quad (16)$$

$$\begin{array}{c} Q(0) = q_{\varsigma\varsigma}(0)+q_{\varsigma\varsigma}(4)+\dots+q_{\varsigma\varsigma}(N-4), \\ Q(1) = q_{\varsigma\varsigma}(1)+q_{\varsigma\varsigma}(5)+\dots+q_{\varsigma\varsigma}(N-3), \\ Q(2) = q_{\varsigma\varsigma}(2)+q_{\varsigma\varsigma}(6)+\dots+q_{\varsigma\varsigma}(N-2), \\ Q(3) = q_{\varsigma\varsigma}(3)+q_{\varsigma\varsigma}(7)+\dots+q_{\varsigma\varsigma}(N-1). \end{array} \quad (17)$$

The system of equations (15) and the next restriction allow the quantities of samples -1 and $+1$ in sums $S(0),S(1),S(2),S(3)$ to be precisely determined. As a result, the computational load during the synthesis of BNPSs can be significantly reduced. This fact will be explained in more detail a bit later.

R 3) *Excluding equivalent signals from the survey*.

This restriction accounts the fact that every PM signal can be transformed in several ways in other PM signals with equivalent auto-correlation properties. From all known equivalent transformations [4, 6], the following five can be exploited in the synthesis of BNPSs.

ET 1) All the cyclically shifted versions $\{\varsigma_{sh(l)}(i)\}_{i=0}^{N-1} = \{\varsigma\langle i+l\rangle\}_{i=0}^{N-1}$, $l = 0,1,\dots,N-1$ of the PM signal (2) have cyclically shifted PAFCs (the positions of the main-lobes of PACFs coincide).

ET 2) The PM signal (2) and its negative version $\{\varsigma_n(i)\}_{i=0}^{N-1} = \{-\varsigma(i)\}_{i=0}^{N-1}$ have equivalent AFCs and PACFs.

ET 3) The PM signal (2) and its reverse version $\{\varsigma_{rev}(i)\}_{i=0}^{N-1} = \{\varsigma(N-1-i)\}_{i=0}^{N-1}$ have equivalent ACFs and PACFs.

ET 4) The alternate version $\{\varsigma_{alt}(i)\}_{i=0}^{N-1} = \{(-1)^i\varsigma(i)\}_{i=0}^{N-1}$ of the PM signal (2) has alternate ACF and alternate PACF if $N \equiv 0 \bmod 2$.

ET 5) The decimated version $\{\varsigma_{k_d}(i)\}_{i=0}^{N-1} = \{\varsigma\langle k_d i\rangle\}_{i=0}^{N-1}$, $k_d \in K_d$ of the PM signal (2) has decimated PACF. Here K_d is the set of all integers, which simultaneously are smaller than N and are co-prime with N.

The usage of the above restrictions in our renewed computer program leads to a significant reduction of the computational load during the synthesis of BNPSs. This result can be explained as follows.

At the beginning it should be seen that from (9) ensues

$$Q(1) = Q(3). \tag{18}$$

Consequently, the system of equations (15) can be simplified to the form

$$\left|\begin{aligned} &[S(0)+S(1)+S(2)+S(3)]^2 = Q(0)+2Q(1)+Q(2) \\ &[S(0)-S(1)+S(2)-S(3)]^2 = Q(0)-2Q(1)+Q(2) \\ &[S(0)-S(2)]^2 + [S(1)-S(3)]^2 = Q(0)-Q(2) \end{aligned}\right. \tag{19}$$

Now it will be proved that

$$Q(1) = Q(3) = 0. \tag{20}$$

For this aim (10) and the fact that PACFs of BNPSs have both the possibly smallest quantity and magnitude of side lobes for every specific length N are accounted in (19). This gives reason the left side of the first equation of (19) to be presented in the form

$$[S(0)+S(1)+S(2)+S(3)]^2 = \begin{cases} N-4K_- = a_l^2, \\ N+4K_+ = a_h^2. \end{cases} \tag{21}$$

In (21) K_-, K_+ and $N-1-K_\mp$ are the quantities of the PACF side lobes $q_{\min} = -4$, $q_{\min} = +4$ and $q_{\min} = 0$ respectively.

Here it should be pointed out that K_- and K_+ are the smallest integers, transforming the quantities $N-4K_-$ and $N+4K_+$ in exact even squares

$a_l^2, a_h^2, 0 \le a_l < a_h$ respectively. For example, let $N = 48$. As all exact even squares in the range [0,100] are

$$4, 16, 36, 64, 100, \tag{22}$$

from (21) two possibilities ensue

$$[S(0) + S(1) + S(2) + S(3)]^2 = a_l^2 = 36 = 48 - 4K_- \Rightarrow K_- = 3, \tag{23}$$

$$[S(0) + S(1) + S(2) + S(3)]^2 = a_h^2 = 64 = 48 + 4K_- \Rightarrow K_+ = 4. \tag{24}$$

Analogously, the left side of the second equation of (19) can be presented in the form

$$[S(0) - S(1) + S(2) - S(3)]^2 = \begin{cases} N - 4(K_{e-} - K_{o-}) = b_1^2, \\ N + 4(K_{e+} - K_{o+}) = b_2^2. \end{cases} \tag{25}$$

In (25) K_{e-}, K_{e+} and K_{o-}, K_{o+} are the quantities of the PACF side lobes $q_{\min} = -4$ and $q_{\min} = +4$, which positions are even and odd numbers respectively. Besides, as $|K_{e-} - K_{o-}| \le K_- = K_{e-} + K_{o-}$ and $|K_{e+} - K_{o+}| \le K_+ = K_{e+} + K_{o+}$ the perfect squares b_1^2, b_2^2 in the right sides of (25) can be a_l^2 or a_h^2 (except for $N = 8, 20, 40, 68$), i.e.

$$b_1^2, b_2^2 \in \{a_l^2, a_h^2\}. \tag{26}$$

Firstly, the case $N - 4K_- = a_l^2$ in (21) will be analysed. In this situation the right sides of the first and second equations of (19) can be viewed as the following system of equations

$$\left| \begin{aligned} Q(0) + 2Q(1) + Q(2) &= a_l^2 \\ Q(0) - 2Q(1) + Q(2) &= b^2,\ b^2 \in \{a_l^2, a_h^2\} \end{aligned} \right. \tag{27}$$

After subtraction of pair-wise sides of (27), the result is

$$Q(1) = \frac{a_l^2 - b^2}{4}. \tag{28}$$

Here the following circumstances have to be taken in consideration. First, $Q(1) \equiv 0 \bmod 4$ as $Q(1) \in \{0, -4, \ldots, -4(K_-/2)\}$ or $Q(1) \in \{0, -4, \ldots, -4(K_-/2) - 2\}$ for $K_- \equiv 0 \bmod 2$ or $K_- \equiv 1 \bmod 2$ respectively. Second, analysis of all pairs of successive perfect squares in (22) shows that $\left(a_l^2 - a_h^2\right)/4 \equiv \pm 1 \bmod 4$. Consequently, (28) can be consistent only if $b^2 = a_l^2$, i.e.

$$Q(1) = \frac{a_l^2 - a_l^2}{4} = 0. \tag{29}$$

The case $N + 4K_+ = a_h^2$ in (21) can be analysed analogously.

The above arguments prove (20) (except for $N = 8, 20, 40, 68$). As a result, during the synthesis of BNPSs the following two simplified variants of the system (19) have to be explored

$$\begin{cases} S(0)+S(1)+S(2)+S(3)=\pm a_l \\ S(0)-S(1)+S(2)-S(3)=\pm a_l \\ [S(0)-S(2)]^2+[S(1)-S(3)]^2=Q(0)-Q(2) \end{cases} \quad (30)$$

$$\begin{cases} S(0)+S(1)+S(2)+S(3)=\pm a_h \\ S(0)-S(1)+S(2)-S(3)=\pm a_h \\ [S(0)-S(2)]^2+[S(1)-S(3)]^2=Q(0)-Q(2) \end{cases} \quad (31)$$

Here three facts should be pointed out. First, the perfect squares a_l^2 and a_h^2 are defined by the specific length N of the synthesized BNPS. Second, $N-4K_- \le Q(0) \le N, |Q(2)/4| \le K_-$ in (30) and $N \le Q(0) \le N+4K_+, |Q(2)/4| \le K_+$ in (31). Third, in both systems (30), (31) $Q(0)-Q(2)$ has to be a sum of two even perfect squares. The last fact can be presented in the form

$$Q(0)-Q(2)=c_l^2+d_l^2,\ c_l \ge 0 \cap d_l \ge 0, \quad (32)$$

$$Q(0)-Q(2)=c_h^2+d_h^2,\ c_h \ge 0 \cap d_h \ge 0. \quad (33)$$

In (32) and (33) the perfect even squares c_l^2, d_l^2 and c_h^2, d_h^2 in systems (30) and (31) respectively are defined by the specific length N of the synthesized BNPS.

Firstly, the system (30) will be analysed. More specifically, the first equation of (30) can be restricted to the case

$$S(0)+S(1)+S(2)+S(3)=a_l \ge 0, \quad (34)$$

as this inequality takes place either for the BNPS $\{\varsigma(i)\}_{i=0}^{N-1}$ or for its negative version $\{\varsigma_n(i)\}_{i=0}^{N-1} = \{-\varsigma(i)\}_{i=0}^{N-1}$, which have equivalent PACFs, according to ET 2.

Analogously, the second equation of (30) can be restricted to the case

$$S(0)-S(1)+S(2)-S(3)=a_l \ge 0, \quad (35)$$

as this inequality takes place either for the BNPS $\{\varsigma(i)\}_{i=0}^{N-1}$ or for its reverse version $\{\varsigma_{rev}(i)\}_{i=0}^{N-1} = \{\varsigma(N-1-i)\}_{i=0}^{N-1}$, which have equivalent PACFs, according to ET 3. Here it should be pointed out that the exchange of a BNPS by its reverse version does not influence over (34). Indeed, for the reverse signal $\{\varsigma_{rev}(i)\}_{i=0}^{N-1}$ the sums (16) are

$$S_{rev}(0)=S(3), S_{rev}(1)=S(2), S_{rev}(2)=S(1), S_{rev}(3)=S(0). \quad (36)$$

At the end it is easy to be seen that the third equation of the system (31) is equivalent either to the system of equations

$$\begin{vmatrix} S(0)-S(2)=\pm c_l \\ S(1)-S(3)=\pm d_l \end{vmatrix} \quad (37)$$

or to the system of equations

$$\begin{vmatrix} S(0)-S(2)=\pm d_l \\ S(1)-S(3)=\pm c_l \end{vmatrix} \quad (38)$$

The first equation of (37) can be restricted to the case

$$S(0)-S(2)=c_l \geq 0, \quad (39)$$

as this inequality takes place either for the BNPS $\{\varsigma(i)\}_{i=0}^{N-1}$ or for its cyclically shifted version $\{\varsigma_{sh(l)}(i)\}_{i=0}^{N-1}=\{\varsigma\langle i+l\rangle\}_{i=0}^{N-1}$, $l=2$, which have cyclically shifted PACFs, according to ET 1. Here it is easy to be seen that the exchange of a BNPS by its cyclically shifted version at two positions ($l=2$) does not influence over (34) and (35).

Analogously, the second equation of (36) can be restricted to the case

$$S(1)-S(3)=d_l \geq 0, \quad (40)$$

as this inequality takes place either for the BNPS $\{\varsigma(i)\}_{i=0}^{N-1}$ or for its decimated version $\{\varsigma_{k_d}(i)\}_{i=0}^{N-1}=\{\varsigma\langle k_d i\rangle\}_{i=0}^{N-1}$, $k_d=3$, which have decimated PACFs, according to ET 5. Here again it is clear that the exchange of a BNPS by its decimated version with $k_d=3$ does not influence over (34), (35) and (39).

Obviously, the above analysis is truly applicable for the system (31).

Consequently, during the synthesis of BNPSs the following four linear variants of the system (19) have to be explored

$$\begin{vmatrix} S(0)+S(1)+S(2)+S(3)=a_l \\ S(0)-S(1)+S(2)-S(3)=a_l \\ S(0)-S(2)=c_l \\ S(1)-S(3)=d_l \end{vmatrix} \quad (41)$$

$$\begin{vmatrix} S(0)+S(1)+S(2)+S(3)=a_l \\ S(0)-S(1)+S(2)-S(3)=a_l \\ S(0)-S(2)=d_l \\ S(1)-S(3)=c_l \end{vmatrix} \quad (42)$$

$$\begin{vmatrix} S(0)+S(1)+S(2)+S(3)=a_h \\ S(0)-S(1)+S(2)-S(3)=a_h \\ S(0)-S(2)=c_h \\ S(1)-S(3)=d_h \end{vmatrix} \quad (43)$$

$$\left|\begin{array}{c} S(0)+S(1)+S(2)+S(3)=a_h \\ S(0)-S(1)+S(2)-S(3)=a_h \\ S(0)-S(2)=d_h \\ S(1)-S(3)=c_h \end{array}\right. \tag{44}$$

It is not hard the solutions of the above linear systems of equations (41) - (44) to be found. As a result the quantities $N_{0-}, N_{1-}, N_{2-}, N_{3-}$ of samples -1 in sums $S(0), S(1), S(2), S(3)$ respectively can be determined, because every of these sums contains $N/4$ samples -1 and $+1$. Here it should be pointed out especially that due to the arithmetic properties of the set (22) in every pair of systems (41), (42) and (43), (44) respectively only one system has solution, which is consistent with the signal length N. Due to the limited space this fact will not be rigorously proven, but it will be illustrated with the above example $N = 48$. Namely, for $N = 48$ it is not hard to verify that the solutions of systems (40) and (41) are respectively:

$$S(0)=6,\ S(1)=2,\ S(2)=0,\ S(3)=-2, \tag{45}$$

$$S(0)=5,\ S(1)=3,\ S(2)=1,\ S(3)=-3. \tag{46}$$

The last solution is inconsistent with the signal length $N = 48$ as $N/4 = 12$ and every sum of even quantity of samples -1 and $+1$ must be an even integer (analogously, every sum of odd quantity of samples -1 and $+1$ must be an odd integer).

The arguments, substantiated in this section, demonstrate that the precise determination of quantities $N_{0-}, N_{1-}, N_{2-}, N_{3-}$ of samples -1 in sums $S(0), S(1), S(2), S(3)$ allows the volume V_{BNPS} of the BNPSs space to be reduced:

$$V_{BNPS} = 2.C_{N/4}^{N_{0-}}.C_{N/4}^{N_{1-}}.C_{N/4}^{N_{2-}}.C_{N/4}^{N_{3-}} << V_{BS} = 2^N. \tag{47}$$

As ordinarily, in (47) C_n^k stands for "n element combinations in k-classes":

$$C_n^k = \frac{n!}{k!(n-k)!}. \tag{48}$$

Besides, $V_{BS} = 2^N$ is the volume of the binary signals space.

3. MAIN RESULTS OF CONDUCTED SURVEY OF PERIODIC BINARY NEARLY PERFECT SIGNALS WITH LENGTHS N≡0MOD4

On the base of all restrictions, substantiated in the previous section of this paper, the universal computer program for search of BNPSs, presented in our paper [9], has been modified and elaborated. Examples of the found BNPSs with lengths $N \equiv 0 \bmod 4,\ 0 < N \leq 100$, which have the smallest losses in the SNR, are presented in Table 1 (the signals from № 1 to № 10 are known from our previous paper [9] and here they are shown only for completeness).

Table 1. Periodic binary nearly perfect signals with lengths N≡0MOD4

№	*N*	*The characteristic set of the binary nearly perfect signal*	*γ*
1	4	{3}	1,0000
2	8	{1 2 4}	1,3334
3	12	{1 2 4 8}	1,1250
4	16	{1 2 3 5 9 14}	1,2589
5	20	{1 2 3 5 9 12 17}	1,1111
6	24	{1 2 7 10 12 16 19 20 23 24}	1,0607
7	28	{1 2 4 6 7 8 11 14 19 20 28}	1,1305
8	32	{1 2 3 4 6 7 10 16 18 20 23 27 28}	1,0950
9	36	{2 3 5 8 9 10 12 16 18 19 20 21 24 28 33}	1,0455
10	40	{1 2 4 5 10 13 17 18 19 20 23 24 25 28 30 32 34}	1,0575
11	44	{1 4 5 8 9 10 12 15 19 20 21 22 24 26 27 30 32 39 40}	1,0958
12	48	{4 5 6 8 9 11 12 14 17 21 27 28 29 31 32 39 41 43 46 47 48}	1,0475
13	52	{2 3 4 6 7 15 17 18 20 22 23 28 29 35 37 38 42 44 46 47 48 51 52}	1,0993
14	56	{2 3 4 5 7 13 14 17 21 22 27 31 33 34 37 38 39 40 42 44 46 47 49 55}	1,1557
15	60	{2 3 4 7 10 18 22 24 25 27 29 33 34 38 39 41 42 43 44 50 51 52 54 56 57 60}	1,0352
16	64	{2 3 4 7 8 15 16 19 21 22 25 26 30 32 34 35 36 37 39 41 42 44 47 50 56 58 59 60}	1,1221
17	68	{1 2 3 4 5 8 11 15 16 19 21 23 24 25 28 32 33 34 37 39 40 42 43 44 46 49 57 59 65}	1,0765
18	72	{5 6 8 9 14 15 16 21 22 23 24 25 26 28 29 34 35 38 39 43 46 47 49 50 52 54 57 59 61 65 71}	1,1516
19	76	{1 2 3 6 7 8 11 12 16 19 20 21 23 24 25 26 27 34 38 41 48 49 51 55 57 58 60 61 63 67 69 71 72 76}	1,0873
20	80	{1 2 5 8 10 13 16 17 18 21 22 24 26 28 30 33 34 35 36 39 40 41 42 43 48 57 58 62 67 68 69 71 72 78 79}	1,1488
21	84	{1 4 5 6 7 8 9 10 11 14 16 17 23 24 26 27 30 31 32 36 37 41 44 48 51 53 55 56 59 60 62 64 65 67 73 75 77 79}	1,2784
22	88	{1 5 7 8 9 11 12 13 14 15 21 23 24 25 27 28 30 31 32 36 37 40 49 51 55 58 60 61 62 66 69 70 72 75 77 80 82 83 87 88}	1,1760
23	92	{1 2 3 7 9 12 17 18 19 22 24 35 37 38 43 44 46 47 48 50 51 55 56 59 60 64 65 67 70 71 74 76 77 78 79 81 85 87 89 91 92}	1,1153
24	96	{8 9 10 11 13 14 17 20 21 23 26 27 28 29 30 34 35 39 42 43 44 46 53 55 57 61 65 70 71 76 77 79 81 82 84 85 86 87 89 92 93 94 96}	1,1273
25	100	{4 5 7 10 14 17 20 21 26 27 28 29 31 36 38 41 47 49 51 53 54 55 56 57 59 61 63 64 65 66 68 69 70 74 77 81 82 83 85 86 88 89 94 99 100}	1,2477

For brevity, in Table 1 the following notation is used.

First, the characteristic sets of the BNPSs, comprising only the positions of the samples -1, are presented in the third column.

Second, $\gamma, \gamma \geq 1$ denotes the coefficient of losses in the SNR, when the respective BNPS with length $N \equiv 0 \bmod 4,\ 0 < N \leq 100$ is processed by its SLSF.

From the results of the conducted survey of BNPSs with lengths $N \equiv 0 \bmod 4,\ 0 < N \le 100$ the following conclusions can be made.

First, the BNPSs with lengths $N \equiv 0 \bmod 4$ have PACFs, which possess large zones with zero-level side-lobes. The correctness of this conclusion is demonstrated on Fig. 1, where the PACF of the BNPS with length $N = 76$ (BNPS № 19 from Table 1) is shown.

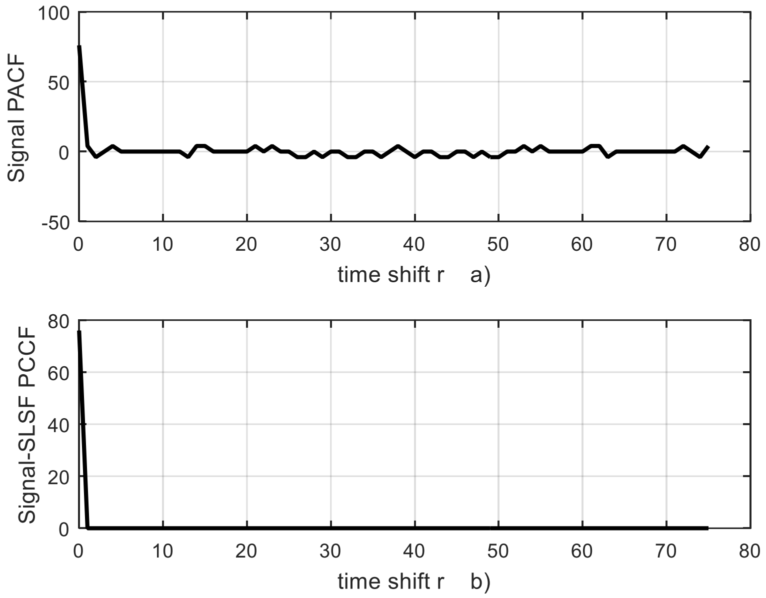

Fig. 1. The PACF (a) of the periodic binary nearly perfect signal № 19 from Table 1 and the PCCF (b) after processing by its SLSF

In fact, the ratio of the quantity of all zero side lobes ($N-1-K_{-}-K_{+}$) to the quantity of all side lobes ($N-1$) of the PACF

$$ZCZR = \frac{N-1-K_{-}-K_{+}}{N-1}, \tag{49}$$

presenting the relative size of the zero correlation zone, varies from 100% (for BNPS № 1) to 37,37% (for BNPS № 25). Besides, the ZCZR for BNPSs, presented in Table 1, is smaller than 50% only for BNPSs № 21, № 23, № 24 and № 25.

Second, the processing of the BNPSs with lengths $N \equiv 0 \bmod 4$ with SLSFs is accompanied with very small losses in the SNR. As a result, the BNPSs with lengths $N \equiv 0 \bmod 4,\ 0 < N \le 100$, found during the survey, presented in the paper, are as beneficial for the radar sensors as the unique Barker signal (4). Indeed, let $R_{\max MF}$ and $R_{\max SLSF}$ be the maximal observation distances of a radar sensor, when its receiver exploits MF and SLSF respectively. It is not hard to prove the relation:

$$R_{\max SLSF} = \sqrt[4]{\frac{1}{\gamma}} R_{\max MF}. \tag{50}$$

Consequently, in the worst case $\gamma = 1{,}2589$ (for BNPS № 4 in Table 1) the maximal observation distance decreases with only $5{,}6\%$, when the radar receiver processes BNPS with its SLSF. This is a very reasonable cost for the providing of the best possible resolution of the detected echo-signals.

Third, most of the BNPSs, presented in Table 1, possess both good (optimal) aperiodic and periodic correlation properties. In fact, the maximal magnitude of ACF side-lobes of these signals do not exceed 6 (it is 6 only for the BNPS № 25).

Forth, all the BNPS, presented in Table 1, and their equivalent signals can be used as building blocks of much larger PM signals. Indeed, every two PM signals with coefficients of losses γ_1 and γ_2, which lengths N_1 and N_2 are coprime, can be multiplied and the product PM signal will have length $N = N_1 N_2$ and coefficient of losses $\gamma = \gamma_1 \gamma_2$ [6].

4. CONCLUSION

In the paper several restrictions, which diminish the computational load in the exploring the BNPSs with lengths N≡0mod4, are substantiated. These restrictions have been implemented in a computer program, which has been used for an exhaustive research of these signals. As a result several unknown up to now periodic binary nearly perfect signals with lengths $N \equiv 0 \bmod 4,\ N > 40$ have been found.

The results, obtained in the paper, could be useful in the process of development of radar sensor and time-synchronization networks.

AKNOWLEDGEMENTS

This paper is supported by the National Scientific Program "Information and Communication Technologies for a Single Digital Market in Science, Education and Security (ICTinSES)", financed by the Ministry of Education and Science of Bulgaria.

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