# A Method for Synthesis of Nearly Ideal Phase Manipulated Signals

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# Introduction

The radio signals, whose periodic auto-correlation functions (ACFs) resemble the Dirac’s delta function, are named ideal, as they have a critical role for many types of radio communication systems (RCSs). More specifically, these signals allow to diminish the negative effects, caused by the multipath spreading of the electromagnetic waves and by the simultaneous asynchronous communication of multiple users, as well as to improve the resolution of the objects, observed by radars.

# Introduction

Due to the positive features of the ideal radio signals, they are extensively researched during the past seventy years. Despite of all efforts of researches in this area many problems are still far from their final solution. In fact, by known methods ideal signals, the lengths of which coincide with the additive or multiplicative group’s cardinality of some finite algebraic field, can be synthesized.

# Introduction

With regard to the restrictions on signal’s length, inherent to classic methods, in the paper a new method for synthesis of phase manipulated (PM) signals with practically ideal periodic auto-correlation functions (PACFs) is substantiated. The positive features of signals, synthesized by the method, are: the lengths of signals are divisors of multiplicative group’s cardinality of arbitrary finite algebraic field and simple practical implementation.

# Basics of Known Methods for Synthesis of Ideal Phase Manipulated Signals

In this paper the attention is focused on the PM signals with ideal PACFs, exploiting only phase signal alphabets with small sizes $N_a \leq 3$, due to their noise resistance and energetic effectiveness.

The analysis of the known methods for synthesis of unimodular PM signals with ideal PACFs shows that they can be described briefly as follows.

The length $N$ of the ideal PM signal $\{s(i)\}_{i=0}^{N-1}$, which has to be synthesized, can be $N \in \{p_a, p_b^n - 1\}$ (1)

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Here $s(i), i = 0, 1, \dots, N - 1$ are the samples, forming the PM signal, $p_a$ is arbitrary prime integer, $p_b = 2, 3, 5, 7$ and $n$ is arbitrary positive integer.

Second, the set

$$I = \{0, 1, \dots, N - 1\} \qquad (2)$$

of all indices of the samples, forming the PM signal, is presented as direct sum of non-intersected classes (subsets):

$$I = C_{cl}(0) \cup C_{cl}(1) \dots \cup C_{cl}(N_a - 1) \qquad (3)$$

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The classes $C_{cl}(k), k = 0, 1, \dots, N_a - 1$, in (3) are formed after applying one of bijective mappings $\Phi_\theta$ of the set $I$ over the multiplicative group of the finite algebraic field $GF(q), q \in \{p_a, p_b^n\}$:

$$0 \to 0 \cup i \xrightarrow{\Phi_\theta} \theta^i, i = 1, 2, \dots, N - 1 \qquad (4)$$

Here $\theta$ is arbitrary primitive element of $GF(q)$.

Third, let the signal alphabet be

$$\{\alpha(l)\}_{l=0}^{N_a - 1} = \{\alpha(0), \alpha(1), \dots, \alpha(N_a - 1)\} \qquad (5)$$

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Typically, the elements $\alpha(l), l = 0, 1, \ldots, N_a - 1$, are complex numbers with equal magnitude as this condition provides the best energetic effectiveness of the transmitters. Consequently: $\alpha(0) = e^{j0} = 1, \alpha(l) = e^{j\psi_l}, j = \sqrt{-1}$,

$$0 \leq \psi_l < 2\pi, l = 0, 1, \ldots, N_a - 1. \qquad (6)$$

The samples of the PM signal, which have to be synthesized, are defined by the rule

$$if\ \theta^i \in C_{cl}(l) \Rightarrow s(i) = \alpha(l),$$
$$i = 0, 1, \ldots, N - 1, l = 0, 1, \ldots, N_a - 1. (7)$$

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As known, the ternary signal $\{s(i)\}_{i=0}^{N-1}$, whose samples are defined by the rule

$$s(i) = \begin{cases} (-1)^{[\nu i + ind_{\theta}(tr\theta^{\nu i})]}, & tr\theta^{\nu i} \neq 0, \\ 0, & tr\theta^{\nu i} = 0, \end{cases}$$

$$N = \frac{q^n - 1}{q - 1}, \;\; q = p^m, \;\; i = 0, 1, \dots, N-1, \qquad (14)$$

is ideal, when: $p \neq 2$ is arbitrary odd prime integer, $m$ is arbitrary positive integer, $n$ is arbitrary positive odd integer and $\nu$ is divisor of $q-1$, coprime with $n$.

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In (14) $\theta$ is arbitrary primitive element of $GF(q^n)$ and $tr\theta^{\nu i}$ is the trace function, mapping elements of $GF(q^n)$ on elements of $GF(q)$. Besides, $ind_\theta\left(tr\theta^{\nu i}\right)$ is the index of $tr\theta^{\nu i}$. This means that if $\theta^k = tr\theta^{\nu i}, 0 \leq k < q^n - 1$ then $ind_\theta\left(tr\theta^{\nu i}\right) = k$ .

The signal (14) has $K_0 = \frac{q^{n-1}-1}{q-1}$ (15)

zero samples. This peculiarity leads to diminishing of transmitters' energetic effectiveness.

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Accounting this drawback, in the sequel the uniform PM signal $\{s'(i)\}_{i=0}^{N-1}$

$$s'(i) = \begin{cases} e^{j0} = 1, if\ s(i) = 0, \\ e^{j\varphi} = x, if\ s(i) = 1, \\ e^{-j\varphi} = x^{-1}, if\ s(i) = -1, \end{cases}$$

$$i = 0, 1, \dots, N-1, \qquad (16)$$

will be analyzed by a computer program, which was developed by the authors especially for this purpose.

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The computer program consists of the following steps.

Step 1) Input of parameters $p \neq 2, m, n$, and $\nu$ of the initial signal (14).

Step 2) Counting the quantities of terms

$$0 \times 0, 0 \times (+1), (+1) \times 0, 0 \times (-1), (-1) \times 0,$$
$$(-1) \times (+1), (+1) \times (-1), (-1) \times (-1), (+1) \times (+1).$$

Step 3) Forming the equations, the solutions of which eliminate all the side lobes of the PACF (13) of the uniform PM signal (16).

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This procedure is performed after accounting that the sum of quantities of $0 \times 0, (-1) \times (+1), (+1) \times (-1)$ is the quantity of 1s, the sum of quantities of $0 \times (+1), (+1) \times 0$ is the quantity of $x$s, the sum of quantities of $0 \times (-1), (-1) \times 0$ is the quantity of $x^{-1} = x^{*}$s and so on. The results for several values of parameters $p, m, n, \nu$ are presented in Table 1 and Table 2.

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Table 1. Equations for Eliminating the Side Lobes of the PACF of the Uniform PM Signal, $p = 3$

| $p, m,$ $n, \nu$ | r | Terms | | | | |
|---|---|---|---|---|---|---|
| | | $x^2$ | $x^{*2}$ | $x$ | $x^*$ | 1 |
| 3, 1, 3, 2 | class 1 | 2 | 1 | 2 | 4 | **4** |
| | class 2 | 1 | 2 | 4 | 2 | **4** |
| 3,1, 5, 2 | class 1 | 15 | 12 | 24 | 30 | **40** |
| | **class 2** | **12** | **15** | **30** | **24** | **40** |

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Table 2. Equations for Eliminating the Side Lobes of the PACF of the Uniform PM Signal, $p = 5$

| $p, m, n, \nu$ | r | Terms | | | | |
|---|---|---|---|---|---|---|
| | | $x^2$ | $x^{*2}$ | $x$ | $x^*$ | 1 |
| 5, 1, 3, 2 | class 1 | 6 | 4 | 3 | 7 | **11** |
| | class 2 | 4 | 6 | 7 | 3 | **11** |
| | class 3 | 5 | 5 | 5 | 5 | **11** |
| 5, 1, 5, 2 | class 1 | 120 | 130 | 135 | 115 | **281** |
| | class 2 | 130 | 120 | 115 | 135 | **281** |
| | **class 3** | **125** | **125** | **125** | **125** | **281** |

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Step 4) Solving the balancing equations of groups of equations, obtained on the base of results, presented in Table 1 and Table 2.

The solutions of the balancing equations of groups of equations are:

Case $p = 3, m = 1, n = 3, v = 2, N = 13$

$$x = 2^{-1}(-1 \pm 3^{-\frac{1}{2}}) \pm j\sqrt{\frac{2}{3} \pm 2^{-1}3^{-\frac{1}{2}}}, \ (33)$$

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Case $p = 3, m = 1, n = 5, \nu = 2, N = 121$

$$x = 2^{-1}\left(-1 \pm 3^{-\frac{3}{2}}\right) \pm j\sqrt{\frac{20}{27} \pm 2^{-1}3^{-\frac{1}{2}}}, \qquad (34)$$

Case $p = 5, m = 1, n = 3, \nu = 2, N = 31$

$$x = 2^{-2}\left(-1 \pm 5^{-\frac{1}{2}}\right) \pm j\sqrt{\frac{74}{80} \pm 2^{-3}5^{-\frac{1}{2}}}, \qquad (35)$$

Case $p = 5, m = 1, n = 5, \nu = 2, N = 781$

$$x = 2^{-2}\left(-1 \pm 5^{-\frac{3}{2}}\right) \pm j10^{-\frac{3}{2}}\sqrt{937 \pm 5^{-\frac{3}{2}}}. \qquad (36)$$

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Step 5) Analyzing the obtained results.

Due to the imaginary nature of side lobes of FACFs, they can be completely eliminated in two ways.

W 1) At the end stage of digital signal processing, the imaginary parts of all the samples are rejected.

W 2) The uniform PM signal $\{s'(i)\}_{i=0}^{N-1}$, synthesized according to (16), and its complex conjugated signal $\{s'^{*}(i)\}_{i=0}^{N-1}$ form a complementary pair whose aggregated PACF is ideal.

# Conclusion

In the paper a new method for synthesis of uniform PM signals, whose PACFs side lobes are purely imaginary samples, is substantiated. This allows all the side lobes to be eliminated during the final stage of the digital signal processing. Besides, the uniform PM signals, synthesized by the method, have lengths, which form a dense and infinite set. As a result, the number of possible variants in the RCS's development increase significantly, which provides to engineers more possibilities to optimize parameters of RCSs.

Thanks for Your Attention!

Questions?