Multiobjective forecasting model based on the interval discrete type-2 fuzzy sets and genetic algorithm

Liliya Anatolievna Demidova[1] and Maksim Anatolievich Stepanov[2]

[1]Department of Intelligent Systems of Information Security, Institute of Integrated Safety and Special Instrumentation

Russian Technological University – MIREA

Vernadsky Avenue,78, Moscow, 119454, Russia

liliya.demidova@rambler.ru

[2]Department of Computational and Applied Mathematics, Faculty of Computing Engineering

Ryazan State Radio Engineering University named after V.F. Utkin

Gagarin Str., 59/1, Ryazan, 390005, Russia

smastefan@gmail.com

FOU(FOOTPRINT OF UNCERTAINTY) FORECASTING MODEL

We consider one-factor forecasting models. These models involve working with the values of the elements of one TS, which represents some factor. We develop and study the multiobjective *FOU*-model (forecasting model based on the *FOU* of the interval discrete type-2 fuzzy sets (IDT2FS)) and the genetic algorithm for implementing the short-term forecasting for one step forward. We use the average forecasting error (AFER) and the tendency mismatch indicator (TMI) as the criteria. Both criteria should be minimized. They allow to assess the similarity of the predicted values of the known elements of the TS with the real ones, but using the different estimation principles. AFER allows to assess the similarity of the predicted and real values of the known elements of the TS, and criterion TMI allows to assess the similarity of the directions of the change of the predicted and real values of the known elements of the TS.

To solve the problem of optimizing the parameter values of the multiobjective *FOU*-model according to two criteria, various approaches can be applied, including approaches which implement the multiobjective optimization using the GA. We to develop a version of the multiobjective GA with borrowing ideas common to many multiobjective GAs to identify the Pareto-front solutions.

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Let be $d(t)$ ($t=\overline{0,m}$) is the TS which contains the values of elements of some factor, let be $\Delta d(t)$ ($t=\overline{1,m}$) is the TS which contains the increment values of this factor [5, 6]: $\Delta d(t)=d(t)-d(t-1)$. Using the increment values of the factor allows to increase the accuracy of the developed forecasting model.

Let be X is the universe for the $\Delta d(t)$:

$$X=[D_{min}-D_1,\ D_{max}+D_2], \qquad (1)$$

where D_{min}, D_{max} are the minimum and maximum values of the $\Delta d(t)$ ($D_{min}=\underset{t=\overline{1,m}}{min}(\Delta d(t))$, $D_{max}=\underset{t=\overline{1,m}}{max}(\Delta d(t))$);

D_1, D_2 are the real numbers, the use of which allows to divide the universe X into the n intervals of the equal length: x_1, x_2,…, x_n [5, 6].

The IDT2FS $\widetilde{A}$ at the universe X can be written as:

$$\widetilde{A}=u_{\widetilde{A}}(x_1)/x_1+u_{\widetilde{A}}(x_2)/x_2+...+u_{\widetilde{A}}(x_n)/x_n, \qquad (2)$$

where $u_{\widetilde{A}}(x)=\underline{u}_{\widetilde{A}}(x)$, $\overline{u}_{\widetilde{A}}(x)$; $\underline{u}_{\widetilde{A}}(x)$, $\overline{u}_{\widetilde{A}}(x)$ are the "lower" and "upper" membership functions (MF) of the IDT2FS, which define FOU; $u_{\widetilde{A}}(x): X\to[0,1]$; $u_{\widetilde{A}}(x_r)$ ($r=\overline{1,n}$) is the value of the membership of the interval $x_r\in X$ for "lower" and "upper" MFs of the IDT2FS.

$FOU_r=\Lambda/\widetilde{A}_{r-1}+1/\widetilde{A}_r+\Lambda/\widetilde{A}_{r+1}$ ($\Lambda=\alpha_{lower},\alpha_{upper}$), if the increment value belongs to the interval x_r ($r=\overline{2,n-1}$).

$FOU_1=1/\widetilde{A}_1+\Lambda/\widetilde{A}_2$ ($FOU_n=\Lambda/\widetilde{A}_{n-1}+1/\widetilde{A}_n$), if the increment value belongs to the interval x_1 (x_n).

We can write the fuzzy logic dependency (FLD) of the first order for the t-th and $(t+1)$-th time moments: $FOU_j\to FOU_i$, where FOU_j и FOU_i. The FLD of the k-th order can be formed in a similar way.

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The groups of the FLDs (FLDGs) can be determine by combining the FLDs with the same left-hand side into one group. For example, we can be form the FLDG:

$FOU_{j_k}, FOU_{j_{(k-1)}}, \ldots, FOU_{j_1} \rightarrow FOU_{l_1}, FOU_{l_2}, \ldots, FOU_{l_g}$.

The new FOU can be defined as the union of the FOU_{l_k} ($k = \overline{1, g}$) from the right-hand side of the FLDG.

The "lower" and "upper" MFs of the IDT2FS for $\Lambda = \alpha_{lower}, \alpha_{upper}$ and $r = \overline{1, n}$ for the right-hand sides of the FLDG can be calculated as [5, 6]:

$$u_{FOU}(x_r) = max(u_{FOU_{l_1}}(x_r), u_{FOU_{l_2}}(x_r), \ldots, u_{FOU_{l_g}}(x_r)). \quad (3)$$

The FOU-model works with the FLDGs.

When forecasting the value of the TS element for the $(t+1)$-th time moment, the corresponding FOU is determined. The forecasting value $f(t+1)$ for the $(t+1)$-th time moment can be calculated as the sum of the known value $d(t)$ for the t-th time moment and the centroid value, which can be defined as the defuzzified increment value of the $y(t+1)$ for the $(t+1)$-th time moment. The centroid value $y(t+1)$ for the $(t+1)$-th time moment can be calculated with the type-reduction operation. A herewith, the Karnik-Mendel iterative algorithm can be used [4]. We can find the crisp interval $[y_{left}, y_{right}]$, the boundaries' values of which can be used to calculate the crisp value $y(t+1)$ for the $(t+1)$-th time moment: $y(t+1) = (y_{left} + y_{right})/2$.

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It is necessary to note that the special attention should be paid to the correct choice of the parameter values for n, D_1 and D_2, appearing in (1) and (2), as well as to the choice of the FLDGs order. The values of these parameters should be chosen so that there are the non-empty right-hand sides of the FLDGs (including last time moment). A herewith, the forecasting model must satisfy the certain optimality criteria. We propose to use as these criteria the average relative forecasting error and the trend mismatch indicator [7]:

$$AFER = \frac{\sum_{t=1}^{m} \left| (f(t) - d(t))/d(t) \right|}{m} \cdot 100\%, \quad (4)$$

$$Tendency = \frac{h}{m-k-1}, \quad (5)$$

where h is the number of the negative productions $(f(t-1) - f(t)) \cdot (d(t-1) - d(t))$ $(t = \overline{k+2, n})$; $f(t)$ and $d(t)$ are the predicted and real values for the t-th time moment; m is the number of the time moments; k is the order of the forecasting model; $m-k-1$ is the general number of productions $(f(t-1) - f(t)) \cdot (d(t-1) - d(t))$.

Both of these criteria should be minimized.

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Criteria (4) and (5) allow to assess the similarity of the predicted values of the known elements of the TS with the real ones, but using the different estimation principles. Criterion (4) allows to assess the similarity of the predicted and real values of the known elements of the TS, and criterion (5) allows to assess `the similarity of the directions of the change of the predicted and real values of the known elements of the TS.

The optimal values of the parameters of the FOU-model can be found using the genetic algorithm (GA). The chromosome which determines the decision can be encoded as

$s = (D_1, D_2, n, k, \alpha_{lower}, \alpha_{upper})$. (6)

We suggest to define the following boundaries for the chromosome elements: $[0; dl_1]$ for D_1; $[0; dl_2]$ for D_2; $[2; n_{max}]$ for n; $[0;1]$ for α_{lower} and α_{upper}; $[2; k_{max}]$ for k, where dl_1, dl_2 are the positive real numbers; $dl_i = D_{max} - D_{min}$, i=1,2; n_{max} is the natural number, $n_{max} \leq m-1$; m is the number of the time moments; k_{max} is the natural number, $k_{max} \leq m-1$.

APPROBATION OF THE FOU-MODEL

Approbation of the proposed forecasting models was carried out for the example of the TSs group characterizing the indicators of the socio-economic development of the Ryazan region (Russia). Not only the forecasting models themselves and the forecasting results for one step forward were obtained, but also the “corridors” characterizing the evolution of the analyzed TSs at the various moments in time. The form of “corridors” made it possible to draw the additional conclusions about the features hidden in the numerical data.

In addition, according to the results of implementation of the multiobjective GA, in the process of development of the FOU-models, in each case, the construction of the Pareto-front of solutions was implemented with its subsequent visualization at the end of the calculations (Fig. 1).

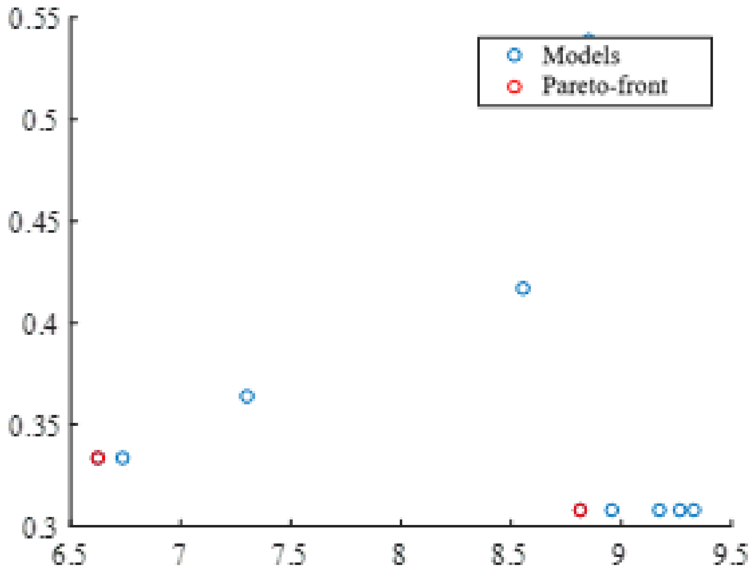

Figure 1 – Models and their criterion values

APPROBATION OF THE FOU-MODEL

The “corridor” for “Monthly average nominal accrued wages in the region (in rubles)” is the narrowest, and for the “Agricultural products (in mln. rubles)”the corridor is the widest. The corridor for the “Gross regional product at basic prices of the corresponding years (in rubles)” occupies an intermediate position between the two above in width (as well as in shape). Therefore, the forecasting model for the first TS (fig. 2) is more accurate than for the third (fig. 4).

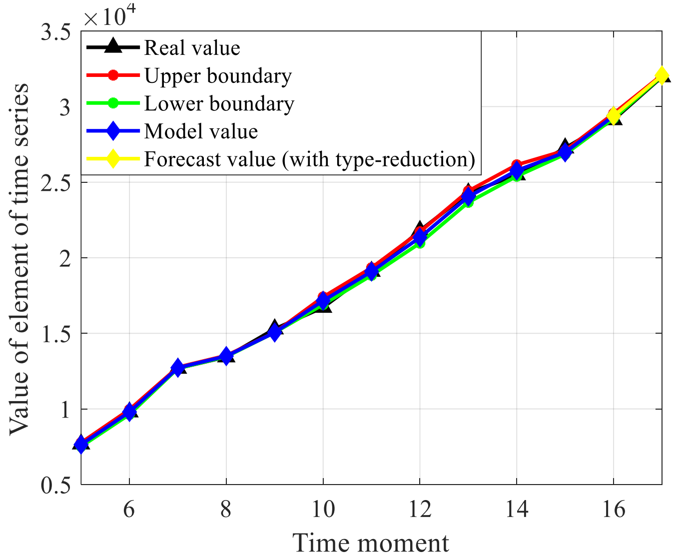

Figure 2 – The results of the model development and forecasting for the “Monthly average nominal accrued wages in the region (in rubles)”

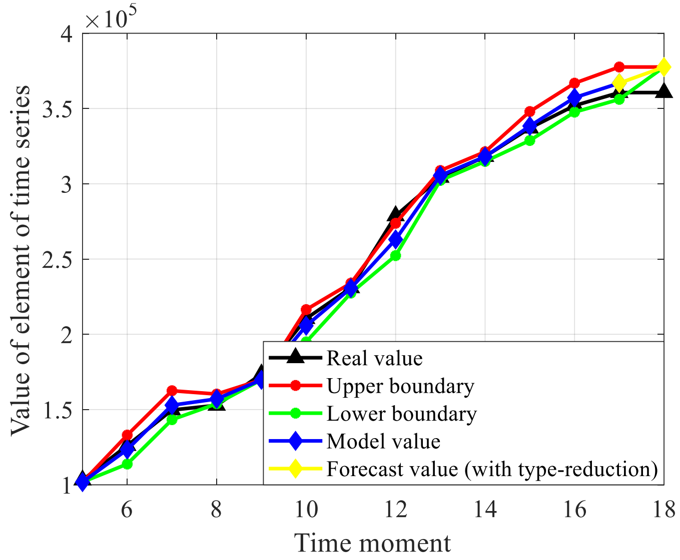

Figure 3 – The results of the model development and forecasting for the “Gross regional product at basic prices of the corresponding years (in rubles)”

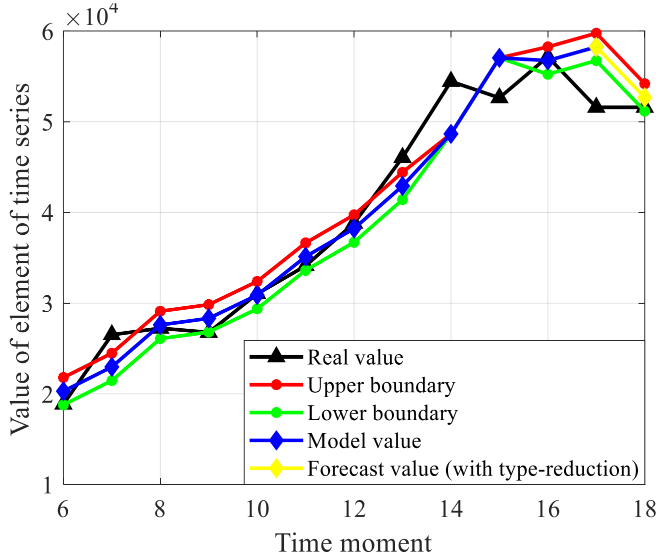

Figure 4 – The results of the model development and forecasting for the “Agricultural products (in mln. rubles)”

APPROBATION OF THE FOU-MODEL

For the first TS (fig. 2), the criteria values are the following: *AFER*= 0.959 %, *Tendency*=0. A herewith, the forecasting error for one step forward equals to 0.279%.

For the second TS (fig. 3), the criteria values are the following: *AFER*= 1.739 %, *Tendency*=0. A herewith, the forecasting error for one step forward equals to 4.703%.

For the third TS (fig. 4), the criteria values are the following: *AFER*= 6.034 %, *Tendency*=0.364. A herewith, the forecasting error for one step forward equals to 2.108%.

CONCLUSION

The proposed *FOU*-model provides the effective solution for the forecasting task of the short TSs for one step forward. This *FOU*-model gives the good visualization of the model development process and allows to visually see the features of the TS evolution at the different points in time and evaluate its variability in terms of the fuzzy set theory. The proposed *FOU*-model is universal in the sense that it covers cases of the T1FS and T2FS due to the possibility of varying the parameter values. This FOU-model can be applied for forecasting the TSs with a short relevant part, in particular, for forecasting the TSs of the socio-economic sphere.