Odessa Polytechnic State University
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Theoretical Foundations of Fractal Electrotechnic. Fractal Elements and its Properties

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Electrical Engineering - From William Hilbert to the Present

- In 1600 W. Gilbert's work "On the Magnet, Magnetic Bodies and the Great Magnet Earth" was published
- In 1733 Charles Francois Dufay discovered two kinds of electric charge: "glass" and "resin“. Now they are called "positive" and "negative“
- In 1759 Academician Franz Ulrich Maria Epinus first noted the existence of a connection between electrical and magnetic phenomena
- In 1785 Charles Coulomb invented a torsion balance and established the law of interaction of two electrified bodies - "Coulomb's law"
- In 1800 Volta invented a source of galvanic current, called the "voltaic pillar“
- In 1802 Davy heated the platinum wire to white heat by passing an electric current through it
- In 1820 Ampere introduced into science the concept of the direction of an electric current and established the law of interaction of currents.

William Gilbert
24.05.1544 – 30.11.1603

Charles Francois Dufay
14.09.1698 – 16.06.1739

Franz Ulrich Maria
Theodor Epinus
13.12.1724 – 10.08.1802

Charles Coulomb
11.06.1736 – 23.08.1806

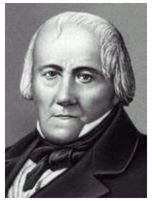
Alessandro Giuseppe
Antonio Anastasio Volta
18.02.1745 – 05.0.1827

Humphry Davy
17.12.1778 – 29.05.1829

Andre-Marie Ampere
20.01.1775 – 10.06.1836

Electrical Engineering - From William Hilbert to the Present

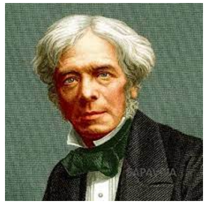
- In 1820 Jean-Baptiste Biot and Felix Savart established the law of interaction between current and magnetic field
- In 1827 Georg Ohm published his work "Galvanic Circuit Mathematically Developed by Dr. RS Ohm", in which he formulated the basic proposition known today as Ohm's Law
- In 1831 Faraday discovered the phenomenon of electromagnetic induction
- In 1873 Maxwell published "A Treatise on Electricity and Magnetism", where he outlined his electromagnetic theory of light
- In 1855 in the article "On Faraday's Lines of Force", Maxwell first wrote down in differential form the system of equations of electrodynamics
- In 1879 Werner von Siemens first uses the term "electrical engineering" in a letter to Heinrich von Stefan, General Postmaster of Germany
- In 1888 M. O. Dolivo-Dobrovolsky invented a three-phase current system



Jean-Baptiste Biot
21.04.1774 – 03.02.1862

Felix Savart
30.06.1791 – 16.03.1841

Georg Simon Ohm
16.03.1787 – 07.07.1854



Michael Faraday
22.09.1791 – 25.08.1867

James Clerk Maxwell
3.06.1831 – 05.11.1879

Werner von Siemens
13.12.1816 – 06.12.1892

Mikhail Osipovich
Dolivo-Dobrovolsky
21.12.1861 – 03.11.1919

Electrical Engineering - From William Hilbert to the Present

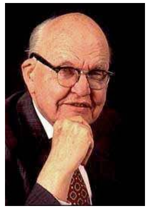
- In 1889 M.O.Dolivo-Dobrovolsky invented a three-phase transformer and a three-phase asynchronous electric motor
- In 1891 M.O.Dolivo-Dobrovolsky built the first three-phase power transmission line with a line voltage of 15,000 V and a capacity of about 200 kW at a distance of 170 km (Lauffen - Frankfurt am Main)
- In 1896 Alexander Popov handed over the world's first wireless message
- In 1907 Boris Rosing invented "a method of electrical transmission of images over a distance" - a television transmitter with mechanical scanning and a television receiver with a cathode ray tube
- In 1941 Konrad Zuse presented the “Z3”, the world's first fully functional and programmable computer
- In 1958 the first integrated circuits were the hybrid integrated circuit invented by Jack Kilby at Texas Instruments

Mikhail Osipovich Dolivo-Dobrovolsky
21.12.1861 – 03.11.1919

Alexander Stepanovich Popov
16.03.1859 – 13.01.1906

Boris Lvovich Rosing
23.04.1869 – 20.04.1933

Konrad Zuse
22.09.1910 – 18.12.1995



Jack St. Clair Kilby
08.11.1923 – 20.06.2005

Electrical engineering - what can we do and what do we know?

Our experience and practical results in electrical engineering are undeniable

But what do we know about electricity?

An optimist will say "a lot"
The realist will say "very little"
The pessimist will say "nothing"

Who are we?

What do we know about…

Electric charge

Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field

There are two types of electric charge: positive and negative

Like charges repel each other and unlike charges attract each other

Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field

This definition describes what happens to the infected matter. But it does not answer the questions: what is a charge and why does it happen

There are two types of electric charge: positive and negative

Why only two kinds?

What is the physical reason for this difference?

Like charges repel each other and unlike charges attract each other

Why do they act this way?

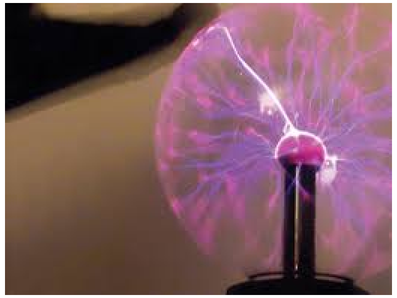
So what do we know about the electric charge?

What do we know about…

Electric field

WIKIPEDIA
The Free Encyclopedia

An electric field is the physical field that surrounds electrically-charged particles and exerts force on all other charged particles in the field, either attracting or repelling them



Electric charge is the physical property of matter that causes it to experience a force when placed in an electromagnetic field

An electric field is the physical field that surrounds electrically-charged particles and exerts force on all other charged particles in the field, either attracting or repelling them

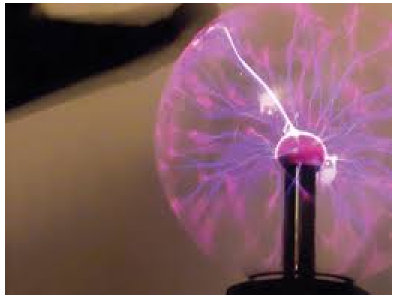
It's practically the same definition, isn't it?

So what do we know about the electric fied?

What do we know about…

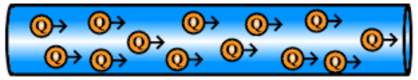
Electric current

An electric current is a stream of charged particles, such as electrons or ions, moving through an electrical conductor or space



An electric current is a stream of charged particles, such as electrons or ions, moving through an electrical conductor or space

If so then



Current = Rate of Flow of Charge 'Q' in a Conductor

Why can an electric current act in a vacuum capacitor, where there are no electric charges?

Why is the speed of propagation of an electric current 100,000 kilometers per second,

while the average speed of an electron in a metal is only 1 centimeters per second?

The turtle is moving faster!

So what do we know about the electric current?

The foundation of modern electrical engineering

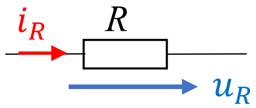
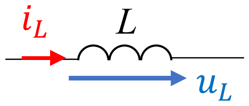
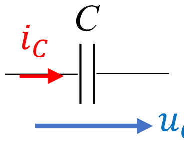
1. There are three main properties of electrical circuits:

- conductivity
- inductance
- capacity

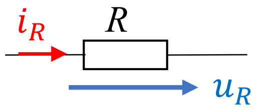
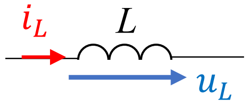
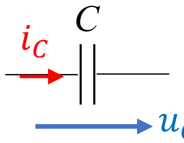
2. These properties are modeled using three ideal elements of electrical circuits:

- resistor
- inductor
- capacitor

3. Relationship between currents and voltages of ideal elements are:

Element	$u = f(i)$	$i = f(u)$
	$u_R = R \cdot i_R$	$i_R = G \cdot u_R$
	$u_L = L \cdot \frac{di_L}{dt}$	$i_L = \frac{1}{L} \int u_L dt$
	$u_C = \frac{1}{C} \int i_C dt$	$i_C = C \cdot \frac{du_C}{dt}$

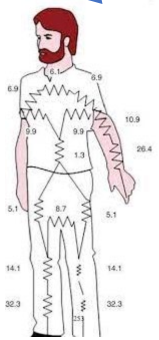
Theoretical electrical engineering. Attempt to generalize

Element	$u = f(i)$	For $i(t) = I_m sin(\omega t + \psi_i)$	Generalization
	$u_R = R \cdot i_R$	$u_R = R \cdot \frac{d^0 i_L}{dt^0}$	$u = \Lambda \frac{d^\alpha i}{dt^\alpha}$
	$u_L = L \cdot \frac{di_L}{dt}$	$u_L = L \cdot \frac{d^1 i_L}{dt^1}$	
	$u_C = \frac{1}{C}\int i_C dt$	$u_C = \frac{1}{C} \cdot \frac{d^{-1} i_L}{dt^{-1}}$	

A generalized fractal element with a parameter describes the properties of all three traditional ideal elements

Theoretical electrical engineering. Attempt to generalize

Application of fractal electrical elements



Human Skin Impedance Simulation for Medical Diagnostics

Ultrahigh frequency communication devices

Fractal materials for stealth technology

Supercapacitors

Extra high capacity batteries

Theoretical electrical engineering. Attempt to generalize

Particular cases of the creation and use of fractal electrical elements have been studied in some detail:

1. L. Lazareck, G. Verch, and A. Peter, "Fractals in circuits," Canadian Conference on Electrical and Computer Engineering, 2001, doi: 10.1109/ccece.2001.933750.
2. E. Akkermans, J. Chen, G. Dunne, L. Rogers, and A. Teplyaev, “Fractal AC Circuits and Propagating Waves on Fractals,” Analysis, Probability and Mathematical Physics on Fractals, 2020, pp.557-567.
3. S. Fujimori, “Analysis of electric discharge with fractal dimension,” Conference on Electrical Insulation & Dielectric Phenomena, 1984, pp. 374-380, doi: 10.1109/EIDP.1984.7684013.
4. M. Nakagawa, and K. Sorimachi, “Basic Characteristics of a Fractance Device,” IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences, vol. E75-A, No.12, 1992, pp. 1814-1819.
5. J. Arnaud, “An introduction to fractals and their applications in electrical engineering,” Journal of the Franklin Institute, vol. 331, issue 6, pp. 659-680, 1994, doi: 10.1016/0016-0032(94)90085-X
6. N. Engheia, "On the role of fractional calculus in electromagnetic theory," in IEEE Antennas and Propagation Magazine, vol. 39, no. 4, pp. 35-46, 1997, doi: 10.1109/74.632994.
7. M. Sugi, Y. Hirano, Y. Miura, and K. Saito, “Simulation of Fractal Immittance by Analog Circuits: An Approach to the Optimized Circuits” IEICE TRANSACTIONS on Fundamentals of Electronics, Communications and Computer Sciences, E82-A(8), 1999, pp.1627-1635.
8. J. Tenreiro Machado, I. Jesus, and A. Galhano, "A Fractional Calculus Perspective in Electromagnetics," 5th International Conference on Multibody Systems, Nonlinear Dynamics, and Control, vol. 6, 2005, pp. 1573-1579, doi: 10.1115/detc2005-84862.
9. V. Tarasov, “Electromagnetic field of fractal distribution of charged particles,” Physics of Plasmas, vol. 12, no. 8, 2005, pp. 082-106, doi: 10.1063/1.1994787.

But the general theory of electrical circuits, which are built from fractal elements, is currently absent.
This research is the first step towards creating such a generalized theory of electrical circuits

Properties of the fractal elements

Fractal derivative of a sinusoidal function

$$u = \Lambda \frac{d^\alpha i}{dt^\alpha} = \Lambda \frac{d^\alpha}{dt^\alpha}[I_m \cdot sin(\omega t + \psi_i)] = \Lambda \cdot \omega^\alpha \cdot I_m \cdot sin\left(\omega t + \psi_i + \frac{\pi}{2}\alpha\right)$$

Special cases:

1. $\alpha = 0 \quad \rightarrow \quad u = \Lambda \cdot I_m \cdot sin(\omega t + \psi_i)$. Corresponds to the classic resistor, $\Lambda = R$
2. $\alpha = +1 \quad \rightarrow \quad u = \Lambda \cdot \omega \cdot I_m \cdot sin\left(\omega t + \psi_i + \frac{\pi}{2}\right)$. Corresponds to the classic inductor, $\Lambda = L$, $\Lambda \cdot \omega = \omega L = X_L$, phase change is: $\varphi = +\frac{\pi}{2}$
3. $\alpha = -1 \quad \rightarrow \quad u = \Lambda \cdot \omega^{-1} \cdot I_m \cdot sin\left(\omega t + \psi_i - \frac{\pi}{2}\right)$. Corresponds to the classic capacitor, $\Lambda = \frac{1}{C}$, $\Lambda \cdot \omega^{-1} = \frac{1}{\omega C} = X_C$, phase change is: $\varphi = -\frac{\pi}{2}$

Thus, classical electrical engineering can be considered as a special case of a fractal with a fractal set $\{-1, 0, +1\}$

Properties of the fractal elements

Generalized fractal element for complex amplitudes (phasors) of currents and voltages

$$u = \Lambda \cdot \omega^{\alpha} \cdot I_m \cdot sin\left(\omega t + \psi_i + \frac{\pi}{2}\alpha\right)$$

Phasor for voltage u is:

$$\dot{U}_m = \Lambda \cdot \omega^{\alpha} \cdot I_m \cdot e^{j\left(\psi_i + \frac{\pi}{2}\alpha\right)} = \Lambda \cdot \omega^{\alpha} \cdot I_m \cdot e^{j\psi_i} \cdot e^{j\frac{\pi}{2}\alpha} = \Lambda \cdot \omega^{\alpha} \cdot \dot{I}_m \cdot e^{j\frac{\pi}{2}\alpha} = \dot{I}_m \cdot \left(\Lambda \cdot \omega^{\alpha} \cdot cos\frac{\pi}{2}\alpha + j\Lambda \cdot \omega^{\alpha} \cdot sin\frac{\pi}{2}\alpha\right)$$

Electrical impedance of the generalized fractal element:

$$\underline{Z} = \Lambda \cdot \omega^{\alpha} \cdot cos\frac{\pi}{2}\alpha + j\Lambda \cdot \omega^{\alpha} \cdot sin\frac{\pi}{2}\alpha$$

Generalized fractal element has two components: resistive (active) and reactive:

$$Z_R = \Lambda \cdot \omega^{\alpha} \cdot cos\frac{\pi}{2}\alpha$$

$$Z_X = \Lambda \cdot \omega^{\alpha} \cdot sin\frac{\pi}{2}\alpha$$

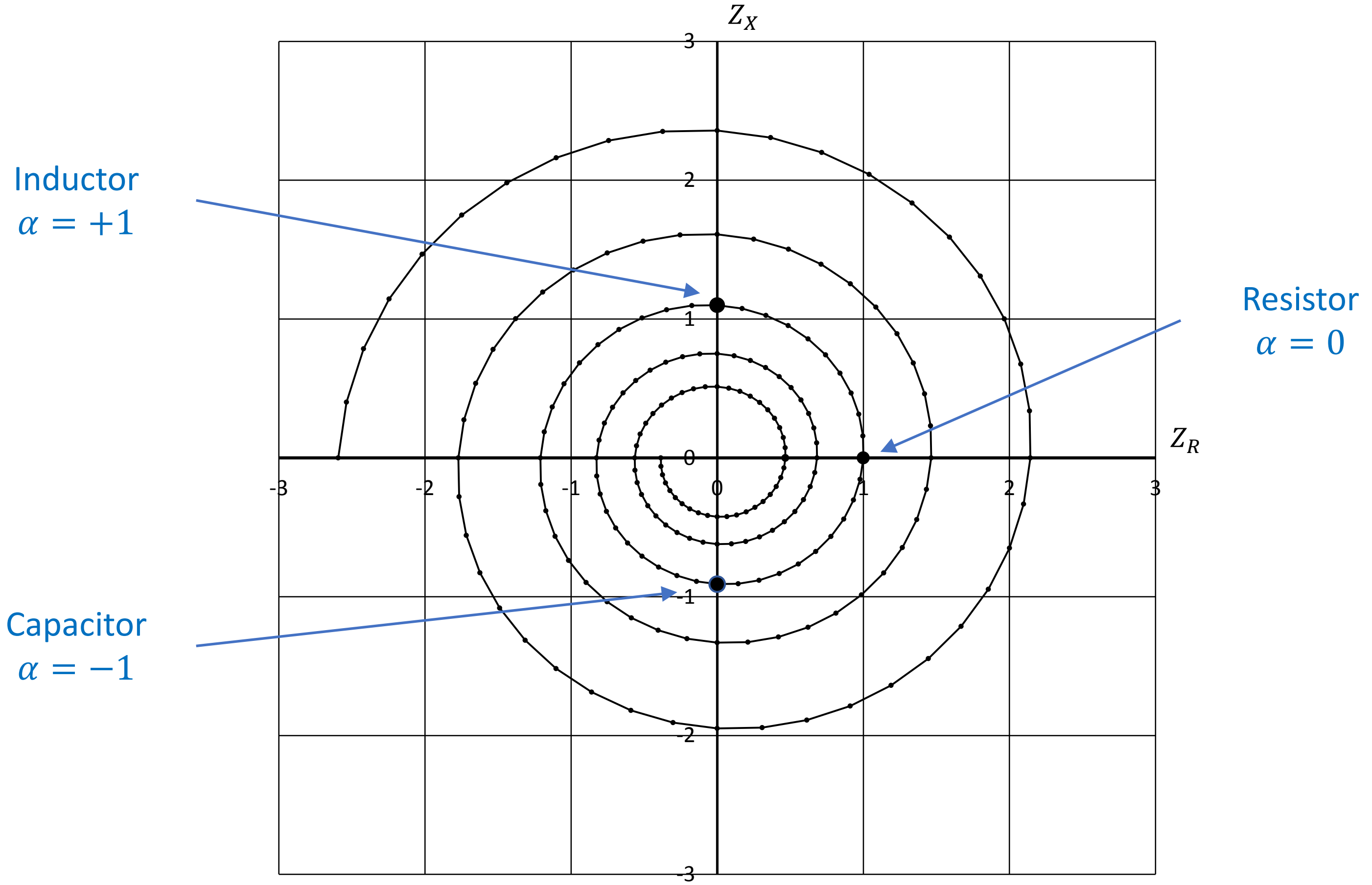
Properties of the fractal elements

Differences between fractal elements and traditional ideal elements

1. Generalized fractal element has two components: resistive $Z_R = \Lambda \cdot \omega^{\alpha} \cdot cos\frac{\pi}{2}\alpha$ and reactive $Z_X = \Lambda \cdot \omega^{\alpha} \cdot sin\frac{\pi}{2}\alpha$. Traditional elements have only either an active or a reactive component
2. The resistive component of traditional elements can only be positive. For fractal elements the active component is positive only in the range $4k - 1 < \alpha < 4k + 1$, where $k \in \mathbb{Z}$ (set of integer)
3. The resistive component of conventional elements is independent of frequency. For fractal elements, the resistive component always depends on the frequency, except for the case when $\alpha = 0$

Properties of the fractal elements

Vector $\underline{Z}$ on the complex plane



Logarithmic spiral

Properties of the fractal elements

Unit of measurement for parameter Λ

Element	Unit of measurement	Fractal order	Dimensional equivalences
Inductance	Henry (H)	$+1$	$m^2 \cdot kg \cdot A^2 \cdot s^{-2}$
Resistance	Ohm (Ω)	0	$m^2 \cdot kg \cdot A^2 \cdot s^{-3}$
Capacity	Farad (F)	-1	$m^2 \cdot kg \cdot A^2 \cdot s^{-4}$
Fractal element	Fractal Ohm $(Fr\Omega)$ or Kirchhoff (Kr)?	α	$m^2 \cdot kg \cdot A^2 \cdot s^{\alpha-3}$

1 Fractal Ohm (or Kirhgoff ?) corresponds:

- 1 H, if $\alpha = +1$
- 1 Ω, if $\alpha = 0$
- 1 F, if $\alpha = -1$

Skin-Effect and fractal elements

As is known, for the classical skin effect, the resistance of a conductor with radius a and with frequency ω is:

$$R = \frac{1}{2\pi a}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{1}{2\pi a}\sqrt{\frac{\mu_0}{2\sigma}} \cdot \omega^{0,5}$$

Where:

μ_0 – is the free space permeability,

σ – is the specific conductivity.

Impedance modulus of the fractal element is:

$$|Z| = \sqrt{Z_R^2 + Z_X^2} = \Lambda \cdot \omega^{\alpha} \cdot \sqrt{cos^2\frac{\pi}{2}\alpha + sin^2\frac{\pi}{2}\alpha} = \Lambda \cdot \omega^{\alpha}$$

The conductor with the classic skin effect is a fractal element with

$$\Lambda = \frac{1}{2\pi a}\sqrt{\frac{\mu_0}{2\sigma}}$$

and a fractal order $\alpha = 0,5$

Conclusions

In this report, an attempt is made to generalize the properties of traditional elements of electrical circuits - a resistor, an inductor and a capacitor for the general case of the fractal dimension of the derivative, which links current and voltage.

The report is purely theoretical in nature, so today the authors are not ready to answer the question "why is it necessary". The resources of Nature and its diversity are boundless, so where exactly the new generalized theory of electrical circuits can find application is difficult to predict. However, already in this article, the fractal properties of an ordinary conductor at a high frequency are noted - the skin effect.

What is behind fractal derivatives? Or is it just beautiful abstract mathematics, or an approach to describing electromagnetic processes in worlds with different physical properties?

After all, the theory of the multiverse (the many-worlds interpretation) by Hugh Everett has not yet been refuted!

Thank you for attention!