ABOUT AUTOMATIC DIFFERENTIAL EQUATIONS IDENTIFICATION BY HYBRID EVOLUTIONARY APPROACH

Karaseva Tatiana

Eugene Semenkin

International Conference on Information Technologies (InfoTech), 2021

DESCRIPTION OF THE APPROACH

Problem statement

Let be a sample of volume n $\{y_i, t_i\}$, i=1, 2,..., n, where y_i is the measurement of the dynamic system output at time t_i. The control action, which is an input of the dynamic system, is known. The object is described by a differential equation with the known initial condition:

$$y^{(k)} = F(y^{(k-1)}, \dots, y', y, x),$$
$$y(0) = y_0$$

The order of the differential equation will be considered to be limited.

It is necessary to build a symbolic model in the form of the differential equation that describes the relationship between the input and output of the system. The initial conditions will be considered to be known.

In this statement, the identification problem is reduced to the problem of symbolic regression solved by **the genetic programming (GP)** algorithm successfully.

DESCRIPTION OF THE APPROACH

Vector's optimization is necessary:

$$(c_1, c_2, \dots, c_n, t_0, \dots, t_k)$$

Constants of the model, obtained by a GP algorithm

For example: $y^k = c_1 y' + c_2 xy + \dots + c_n$.

Initial conditions, obligatory for identification:

$$t_0 = y(x_0) = y_0$$
$$t_1 = y'(x_0) = y'_0$$
$$\dots$$
$$t_k = y^k(x_0) = y_0^k$$

k is the number of initial conditions.

Basic approach

Optimize: $(c_1, c_2, \dots, c_n)$

Main idea:

self-configuring GP + steepest descent method

Hybrid approach

Optimize: $(c_1, c_2, \dots, c_n, t_0, \dots, t_k)$

Main idea:

self-configuring GP + differential evolution (DE)

DESCRIPTION OF THE APPROACH

Self-configuring of GP and DE

Operator groups

	Type selected operators	*Numerical values elected operators*
GP	Selection (Tournament, rank selection, proportionate) Crossover (Standard, single-point, uniform)	Crossover probability Mutation probability
DE	Mutation (Rand1, Rand2, Best1, Best2, Current to Rand, Current to Best)	Differential weight Crossover probability

Algorithms

Population-Level Dynamic Probabilities method

Success History Adaptation algorithm

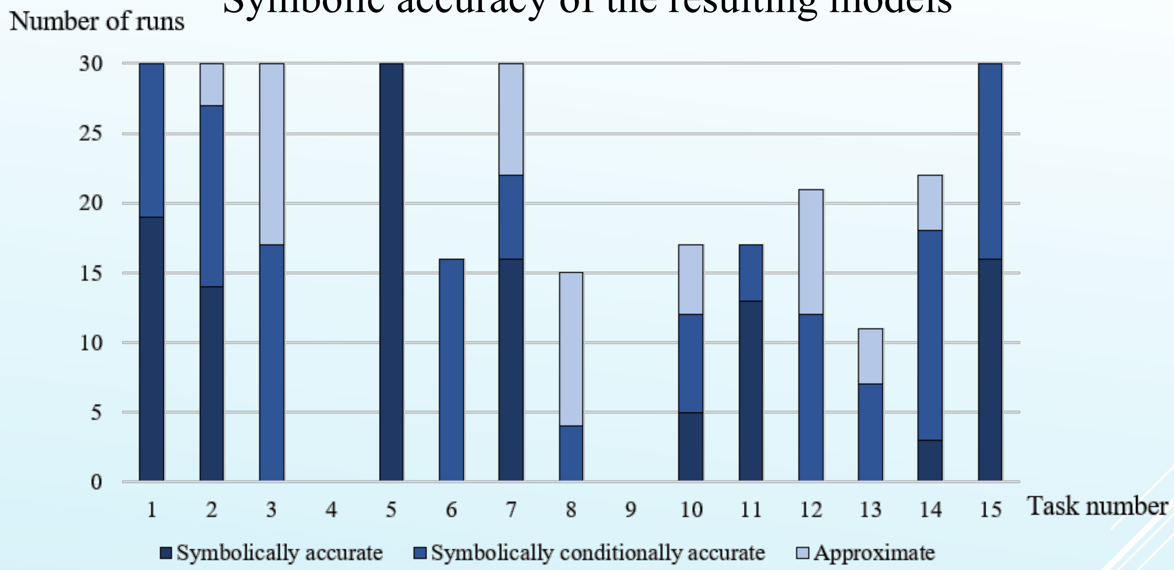
TESTING OF APPROACH

20 arbitrary differential equations of various orders were used to identify differential equations.

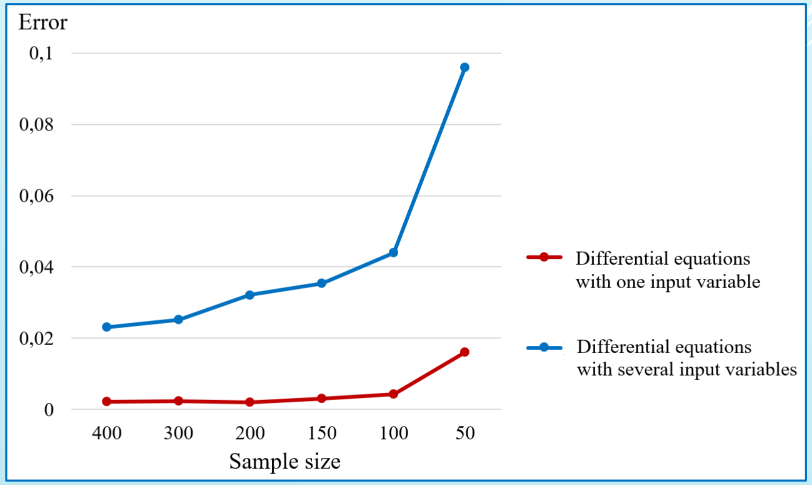
Dependence of the error of the resulted solution under the noise presence in the data

Task number	Error (sample data without noise)	Error (sample data with noise)
1	0.0000	0.0000
2	0.0000	0.0012
3	0.0000	0.0000
4	0.0001	0.0032
5	0.0000	0.0009
6	0.0001	0.0008
7	0.0000	0.0009
8	0.0000	0.0024
9	0.0002	0.0027
10	0.0000	0.0008
11	0.0000	0.0013
12	0.0002	0.0000
13	0.0000	0.0005
14	0.0000	0.0001
15	0.0000	0.0008
16	0.0046	0.0055
17	0.0095	0.0098
18	0.0103	0.0097
19	0.0011	0.0033
20	0.0017	0.0047

Symbolic accuracy of the resulting models



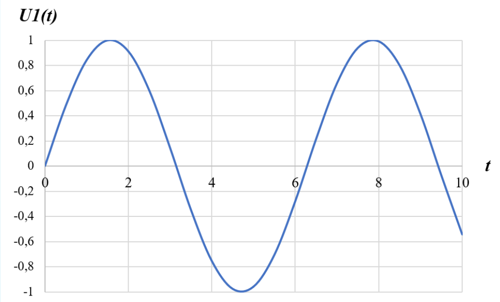
Dependence of the error on the sample size



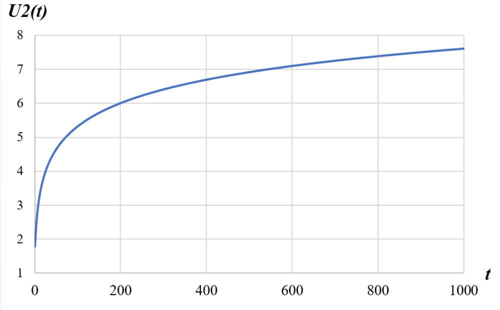
IDENTIFICATION OF DIFFERENTIAL EQUATIONS FOR VARIOUS INPUT EFFECTS

Input effects:

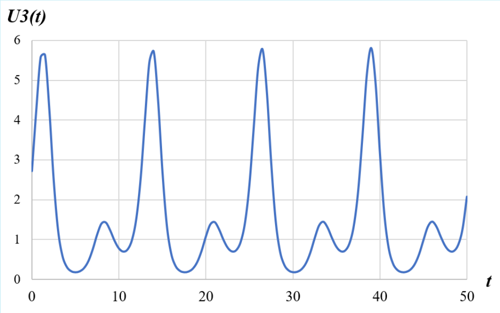
$U1(t) = sin(t)$



$U2(t) = \ln(\sqrt{t^2+1}+5)$

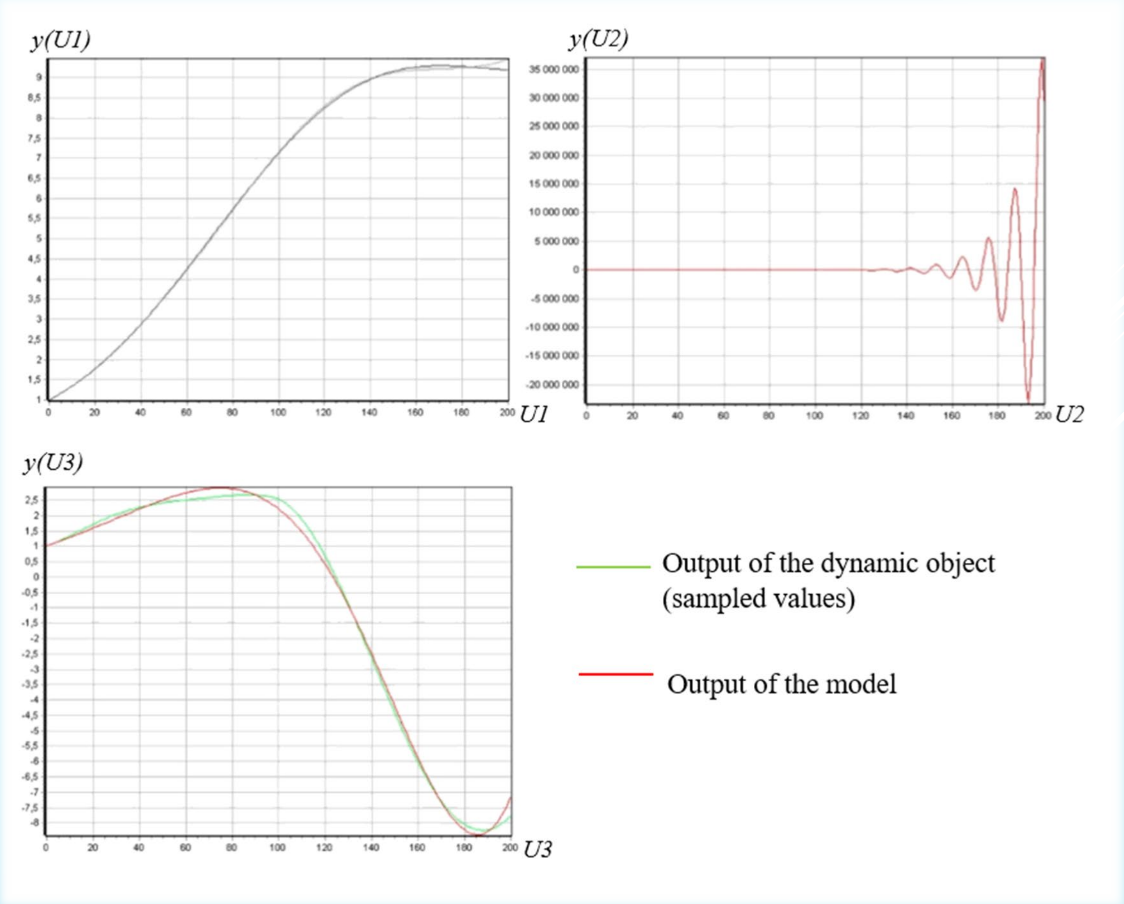


$U3(t) = e^{\sin(t)+\cos(0,5t)}$



Object: $y''' = -y' + \frac{1}{\cos(u)}$

Error ($U1$) = 0.0035, Error ($U2$) = 0.0000, Error ($U3$) = 0.0214



CONCLUSION

- The work presents a hybrid algorithm for the dynamical systems identification, combining a self-configuring genetic programming algorithm for finding the structure of the model in the form of the differential equation and differential evolution method for optimizing numerical parameters of the differential equation and starting point.

- The application of the global optimization method made it possible to significantly reduce an error of the resulting models.

- The further research will be related to modifying the approach for models building in the form of the differential equations system.