Generalizations of the Recursive Methods for Synthesis of Complementary Pairs of Phase Manipulated Signals

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Introduction

The phase manipulated (PM) signals are widely used in many small radar sensors, which are very important elements of the present intelligent factories, smart cars and smart homes. A hard problem in the development of such radar sensors is the simultaneous providing of high object resolution, high accuracy of their spatial coordinates measurement and long range of object detection. A cost-effective approach for solving this problem is the usage of complementary pairs (CPs) of PM signals. Accounting this situation in the paper two general methods have developed, from which all known recursive methods for synthesis of complementary pairs of PM signals follow as particular cases.

Basic Properties of Complementary Pairs of Phase Manipulated Signals

The digital signal processing in the receiver of a radar can be described by the following mathematical model:

$$\left[\sum_{i=0}^{N-1} \mu(i)x^{i}\right]\left[\sum_{i=0}^{N-1} \mu^{*}(i)x^{-i}\right] = \sum_{r=-N+1}^{N-1} P_{\mu\mu}(r)x^{r} \quad (1)$$

In (1) $$F_{\mu}(x) = \sum_{i=0}^{N-1} \mu(i)x^{i} \quad (2)$$

is the so-called generating function or Hall polynomial, associated with the sequence (digital signal) $\{\mu(i)\}_{i=0}^{N-1}$. It consists of N complex numbers, which are the complex envelopes of the elementary phase symbols (or chips) with duration τ_{ch}, forming the echo-signal.

Basic Properties of Complementary Pairs of Phase Manipulated Signals

Analogously, the complex numbers $\mu^*(i), i = 0, 1, \dots, N-1$ (the symbol " $*$ " stands for "complex conjugation") and $P_{\mu\mu}(r), r = -N+1, -N+2, \dots, -1, 0, 1, \dots, N-1$ are the samples of the finite response matched filter (MF), used in the radar receiver, and the autocorrelation function (ACF) of the digital signal $\{\mu(i)\}_{i=0}^{N-1}$ respectively. Besides, the generating functions (Hall polynomials) of the digital signals $\{\mu^*(i)\}_{i=0}^{N-1}$ and $\{P_{\mu\mu}(r)\}_{r=-N+1}^{N-1}$ are

$$F_\mu^*(x^{-1}) = \sum_{i=0}^{N-1} \mu^*(i)x^{-i}, F_{P_{\mu\mu}}(x) = \sum_{r=-N+1}^{N-1} P_{\mu\mu}(r)x^r.$$

Basic Properties of Complementary Pairs of Phase Manipulated Signals

Only the class of the complementary sets of PM signals simultaneously possesses the following properties, which are very important for radar applications:

1) The aggregated ACFs of the complementary sets resemble delta-pulses, providing the maximal possible object resolution and measurement accuracy of object spatial coordinates;

2) The symbol alphabet can have minimal size and symmetric structure;

3) The length N of the complementary sets can be large enough in order to ensure the necessary range of object detection.

Basic Properties of Complementary Pairs of Phase Manipulated Signals

M. Golay and R. Turin have developed recursive methods, which allow CPs of binary PM signals with infinite lengths to be synthesized, using the simplest CPs of binary PM signals with lengths N=1,2 as initial CPs.

Accounting the advancement in the technics for generating and processing of PM signals, as well as the necessity of an efficient exploitation of the electromagnetic spectrum, in the sequel the analysis is focused on the recursive methods for synthesis of CPs, consisting of two PM signals, generated by the means of arbitrary amplitude-phase modulation.

Two Generalizations of the Recursive Methods for Synthesis of Complementary Pairs of Phase Manipulated Signals

Let $\{a(i)\}_{i=0}^{N_a-1}, \{b(i)\}_{i=0}^{N_a-1}$ and $\{c(k)\}_{k=0}^{N_c-1}, \{d(k)\}_{k=0}^{N_c-1}$ be two pairs of arbitrary PM signals. Obviously, these PM signals can be used as building blocks of many derivative PM signals. For example, the derivative signal

$$\{\alpha(l)\}_{l=0}^{N_a N_c-1} =$$

$$= \left\{c(k)\{a(i)\}_{i=0}^{N_a-1}\right\}_{k=0}^{N_c-1} \circledast \left\{d(k)\{b(i)\}_{i=0}^{N_a-1}\right\}_{k=0}^{N_c-1}, \quad (6)$$

where the symbol "$\circledast$" means "concatenation", has length $2N_aN_c$ and it consists of two concatenated parts.

Two Generalizations of the Recursive Methods for Synthesis of Complementary Pairs of Phase Manipulated Signals

The generating function (the Hall polynomial) of the derivative signal (6) is

$$F_\alpha(x) = F_a(x)F_c(x^{N_a}) + x^{N_a N_c} F_b(x) F_d(x^{N_a}). \qquad (7)$$

Let $\{\theta(i)\}_{i=0}^{N-1}$ be an arbitrary sequence. Then, its Hall polynomial is $F_\theta(x) = \sum_{i=0}^{N-1} \theta(i) x^i. \qquad (8)$

Besides, the Hall polynomial of the reversed sequence $\{\theta(N-1-i)\}_{i=0}^{N-1}$ is $\tilde{F}_\theta(x) = \sum_{i=0}^{N-1} \theta(N-1-i) x^i, \quad (9)$

which is the reciprocal polynomial of (8).

Two Generalizations of the Recursive Methods for Synthesis of Complementary Pairs of Phase Manipulated Signals

The main results of this paper are based on the following proposition.

Proposition: Let $\{a(i)\}_{i=0}^{N_a-1}, \{b(i)\}_{i=0}^{N_a-1}$ and $\{c(k)\}_{k=0}^{N_c-1}, \{d(k)\}_{k=0}^{N_c-1}$ be two pairs of arbitrary PM signals. Then the sums of ACFs of the both pairs of derivative PM signals:

Two Generalizations of the Recursive Methods for Synthesis of Complementary Pairs of Phase Manipulated Signals

$$F_\alpha(x) = F_a(x^{N_1})F_c(x^{N_2}) + x^{N_3}F_b(x^{N_1})F_d(x^{N_2}),$$
$$F_\beta(x) = F_a(x^{N_1})\tilde{F}_d^*(x^{N_2}) - x^{N_3}F_b(x^{N_1})\tilde{F}_c^*(x^{N_2}), \quad (11)$$
$$F_\gamma(x) = F_a(x^{N_1})F_c(x^{N_2}) + x^{N_3}F_b(x^{N_1})\tilde{F}_d^*(x^{N_2}),$$
$$F_\delta(x) = F_a(x^{N_1})F_d(x^{N_2}) - x^{N_3}F_b(x^{N_1})\tilde{F}_c^*(x^{N_2}), \quad (12)$$

are product of sums of the initial PM signal ACFs:

$$[F_a(x)F_a^*(x^{-1}) + F_b(x)F_b^*(x^{-1})] \times$$
$$\times [F_c(x)F_c^*(x^{-1}) + F_d(x)F_d^*(x^{-1})]. \quad (13)$$

Two Generalizations of the Recursive Methods for Synthesis of Complementary Pairs of Phase Manipulated Signals

All known at the moment recursive methods for synthesis of CPs of PM signals, presented in:

[4] M. Y. E. Golay, "Complementary series," IRE Transactions on Information Theory, Volume IT–7, Issue 2, pp. 82 – 87, April 1961.
[5] K. Feng, P. Jau-Shyong, and Q. Xiang, "On aperiodic and periodic complementary binary sequences," IEEE Transactions on Information Theory, Volume IT–45, Issue 1, pp. 296 – 303, January 1999.
[6] M. G. Parker, K. G. Paterson, and Ch. Tellambura, "Golay Complementary Sequences," https://www.isg.rhul.ac.uk/~kp/golaysurvey.pdf

could be viewed as particular cases of the proved proposition.

Two Generalizations of the Recursive Methods for Synthesis of Complementary Pairs of Phase Manipulated Signals

Case 1: $N_1 = 1, N_2 = N_a, N_3 = N_a N_c$, every pair $\{a(i)\}_{i=0}^{N_a-1}$, $\{b(i)\}_{i=0}^{N_a-1}$ and $\{c(k)\}_{k=0}^{N_c-1}, \{d(k)\}_{k=0}^{N_c-1}$ is a CP of PM signals.

In this case (11) obtains the form

$$\begin{aligned} F_\alpha(x) &= F_a(x)F_c(x^{N_a}) + x^{N_a N_c} F_b(x) F_d(x^{N_a}), \\ F_\beta(x) &= F_a(x)\tilde{F}_d^*(x^{N_a}) - x^{N_a N_c} F_b(x) \tilde{F}_c^*(x^{N_a}). \end{aligned} \quad (23)$$

(23) is a generalization of the Golay's recursive concatenation method for synthesis of CPs of PM signals, in which the both initial pairs $\{a(i)\}_{i=0}^{N_a-1}$, $\{b(i)\}_{i=0}^{N_a-1}$ and $\{c(k)\}_{k=0}^{c-1}, \{d(k)\}_{k=0}^{N_c-1}$ are CPs of binary signals.

Two Generalizations of the Recursive Methods for Synthesis of Complementary Pairs of Phase Manipulated Signals

Case 2: $N_1 = 1, N_2 = 2N_a, N_3 = N_a$, every pair $\{a(i)\}_{i=0}^{N_a-1}, \{b(i)\}_{i=0}^{N_a-1}$ and $\{c(k)\}_{k=0}^{N_c-1}, \{d(k)\}_{k=0}^{N_c-1}$ is a CP of PM signals. In this case (11) obtains the form

$$\begin{aligned} F_\alpha(x) &= F_a(x)F_c(x^{2N_a}) + x^{N_a}F_b(x)F_d(x^{2N_a}), \\ F_\beta(x) &= F_a(x)\tilde{F}_d^*(x^{2N_a}) - x^{N_a}F_b(x)\tilde{F}_c^*(x^{2N_a}). \end{aligned} \qquad (25)$$

(25) is a generalization of the Golay's block-interleaving recursive method for synthesis of CPs of PM signals, in which the both initial pairs $\{a(i)\}_{i=0}^{N_a-1}$, $\{b(i)\}_{i=0}^{N_a-1}$ and $\{c(k)\}_{k=0}^{N_c-1}, \{d(k)\}_{k=0}^{N_c-1}$ are CPs of binary PM signals.

Two Generalizations of the Recursive Methods for Synthesis of Complementary Pairs of Phase Manipulated Signals

Case 3: $N_1 = 2, N_2 = 2N_a, N_3 = 1$, every pair $\{a(i)\}_{i=0}^{N_a-1}$, $\{b(i)\}_{i=0}^{N_a-1}$ and $\{c(k)\}_{k=0}^{N_c-1}$, $\{d(k)\}_{k=0}^{N_c-1}$ is a CP of PM signals. In this case (11) obtains the form

$$F_\alpha(x) = F_a(x^2)F_c(x^{2N_a}) + xF_b(x^2)F_d(x^{2N_a}),$$
$$F_\beta(x) = F_a(x^2)\tilde{F}_d^*(x^{2N_a}) - xF_b(x^2)\tilde{F}_c^*(x^{2N_a}). \quad (27)$$

(27) is a generalization of the Golay's symbol-interleaving recursive method for synthesis of CPs of PM signals, in which the both initial pairs $\{a(i)\}_{i=0}^{N_a-1}$, $\{b(i)\}_{i=0}^{N_a-1}$ and $\{c(k)\}_{k=0}^{N_c-1}$, $\{d(k)\}_{k=0}^{N_c-1}$ are CPs of binary PM signals.

Two Generalizations of the Recursive Methods for Synthesis of Complementary Pairs of Phase Manipulated Signals

Case 4: $N_1 = 1, N_2 = N_a, N_3 = 0$, the pair $\{c(k)\}_{k=0}^{N_c-1}$, $\{d(k)\}_{k=0}^{N_c-1}$ is a CP of PM signals and the pair $\{a(i)\}_{i=0}^{N_a-1}$, $\{b(i)\}_{i=0}^{N_a-1}$ is obtained from the CP of PM signals $\{\mu(i)\}_{i=0}^{N_a-1}$, $\{\eta(i)\}_{i=0}^{N_a-1}$ by the rules: $F_a(x) = \frac{F_\mu(x)+F_\eta(x)}{2}, F_b(x) = \frac{F_\mu(x)-F_\eta(x)}{2}$.(29). Now (12) obtains the form:

$F_\gamma(x) = F_a(x)F_c(x^{N_a}) + F_b(x)\tilde{F}_d^*(x^{N_a})$, $F_\delta(x) = F_a(x)F_d(x^{N_a}) - F_b(x)\tilde{F}_c^*(x^{N_a})$ (30), which is a generalization of the Turin's symbol-interleaving recursive method.

Conclusion

In the paper two general methods have substantiated, which allow all known recursive methods for synthesis of complementary pairs of PM signals to be presented systematically.

Besides, these general methods can be applied for development of infinitely many types of quasi complementary pairs of PM signals, which can be exploited for preserving working capacity of different types radar sensor networks in hostile radio-electronic environment.