A Method for Synthesis of Mismatched Filters Providing Large Zero Correlation Zones

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Introduction

In the paper a new method for synthesis of mismatched filters for processing the echo-signals in the radar sensors, working in pulse mode, is substantiated. The developed method could be useful for enhancing the performance effectiveness of the radar sensor networks, exploited today widely in all spheres of industry, transportation and civil protection.

Methodology of the Study

Most often, the receivers of the radar sensors are built as digital filters with finite impulse response (FIR), which have the structure, presented on fig. 1.

On fig. 1, the input samples $\{\zeta_{n-M+1-l}\}_{l=0}^{M-1}$ are produced by digitalization of the received signal, which can be noise or so-called echo signal. The last signal is a diminished and phase shifted copy of the signal, emitted by the transmitter (also called “probe pulse”), because the echo signal is product of the reflection of the probe pulse by the object (target) and, besides, it passes two times the distance between the radar and the object (target).

Methodology of the Study

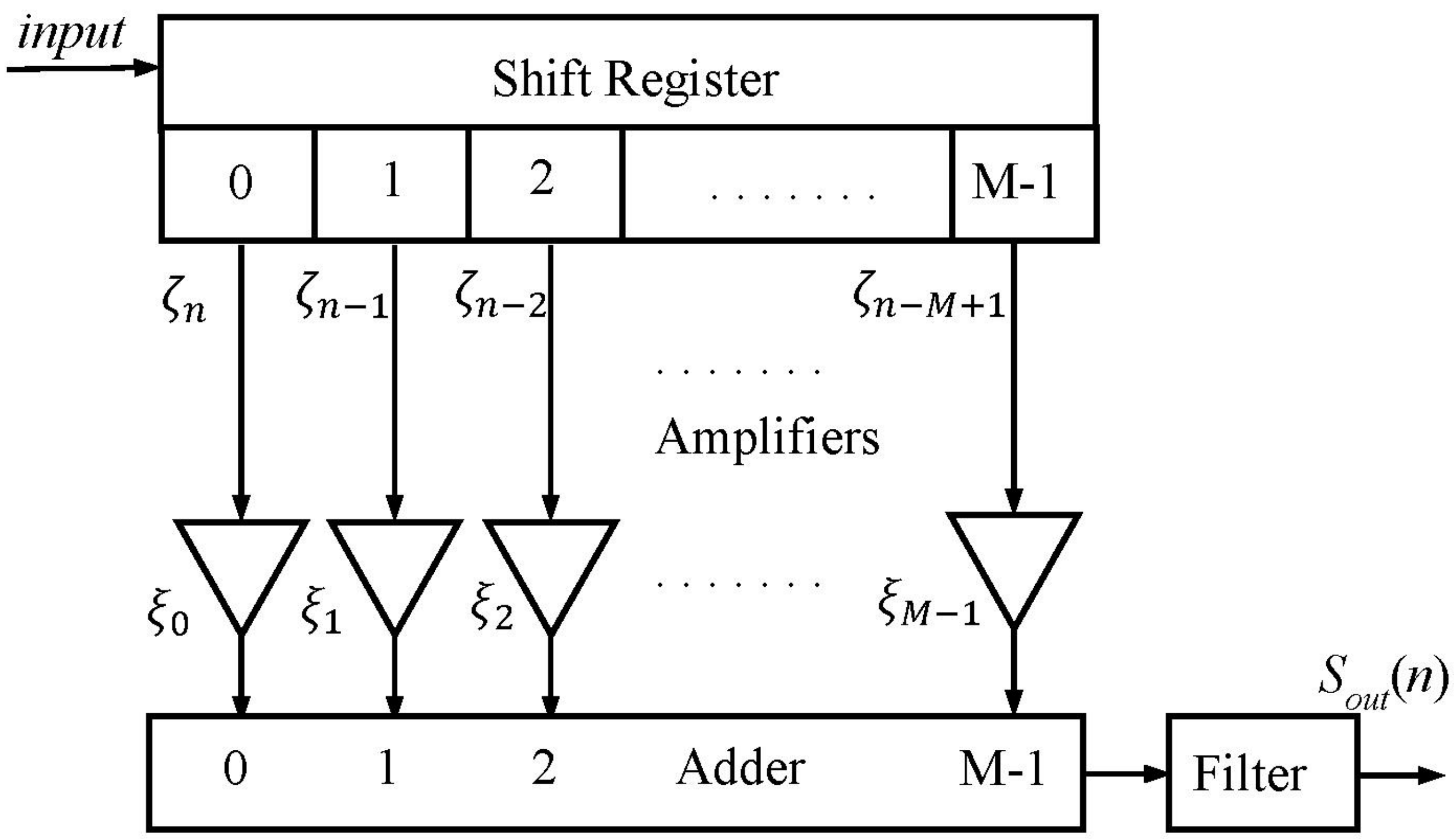

Fig. 1. General block-diagram of the digital filters with finite impulse response

Methodology of the Study

As seen, the output signal (output reaction) of the FIR digital filter (fig. 1) at the n-th clock-interval is:

$$S_{out}(n) = \zeta_n \xi_0 + \zeta_{n-1} \xi_1 + \cdots + \zeta_{n-M+1} \xi_{M-1}. \quad (1)$$

The most often MFs are used in the radar sensors. Such constructive approach is exploited as it maximizes the signal-to-noise ratio (SNR) in the output of the receivers in the presence of additive white Gaussian noise (AWGN), which is the most typical practical situation.

Methodology of the Study

More specifically, let

$$\{\mu(k)\}_{k=0}^{N-1} = \{\mu(0), \mu(1), \dots, \mu(N-1)\} \quad (2)$$

be the phase manipulated (PM) signal, emitted by the transmitter. The samples in the right side of (2) are complex numbers, presenting the complex envelopes of the elementary phase symbols (or chips) with duration τ_{ch}, forming the emitted signal (probe pulse). In this connection, it should be noted that despite of amplitude and phase differences among the emitted signals (probe pulses) and echo signals their autocorrelation properties coincide. Due to this reason, in sequel without loss of generality it is accepted that the received echo signals also are described mathematically by (2).

Methodology of the Study

Typically the samples in the input of the FIR digital filter (fig. 1) are produced by AWGN, which correlation time is smaller than τ_{ch} and its variance and mean square deviation are σ_n^2 and σ_n respectively. In this situation, σ_n is the average amplitude of the AGWN in the input of the receiver and σ_n^2 is its average power. As the amplification is a linear transformation, the samples in the inputs of the adder (fig. 1) have variances $|\xi_l|^2\sigma_n^2 = (\xi_l\xi_l^*)\sigma_n^2 = h_l^2\sigma_n^2, l = 0, 1, \dots, M - 1$. Due to this reason, the average power in the FIR digital filter output is: $W_{nMMF} = \sum_{k=0}^{M-1}|\xi(k)|^2\sigma_n^2 = \sigma_n^2 \sum_{k=0}^{M-1} \xi_k\xi_k^* = E_{MMF}\sigma_n^2$. (3)

Methodology of the Study

In (3) W_{nMMF} is the average power of the MMF output reaction, caused by AWGN, and

$$E_{MMF} = \sum_{k=0}^{M-1} |\xi(k)|^2 = \sum_{k=0}^{M-1} \xi_k \xi_k^* \quad (4)$$

is the so-called MMF's energy.

When the receiver is MF, then $M = N$ (i.e. the lengths of the probe pulse and FIR digital filter (fig. 1) coincide) and its transfer function (TF) is a conjugate and mirror-reverse copy of the probe pulse:

$$\xi_0 = \mu^*(N-1), \xi_1 = \mu^*(N-2), \dots, \xi_{N-1} = \mu^*(0). \ (5)$$

In (5) the symbol "*" stands for "complex conjugation".

Methodology of the Study

After accounting (5) in (4), the result is:

$$\sum_{k=0}^{N-1}|\zeta_k|^2 = \sum_{k=0}^{N-1}|\mu(k)|^2 = \sum_{k=0}^{N-1}\mu(k)\mu^*(k) = U_{ms} \quad . \quad (6)$$

As known, in (6) U_{ms} is the main lobe (peak) of the autocorrelation function (ACF) of the PM signal (2), which is result of coherent accumulation of all its samples.

From (4) and (6) it follows that the average power of the MF's output reaction, caused by AWGN, is:

$$W_{nMF} = \sigma_n^2 \sum_{k=0}^{N-1}\mu(k)\mu^*(k) = \sigma_n^2 U_{ms}. \qquad (7)$$

Methodology of the Study

Now let the FIR digital filter (fig. 1) is MF and, besides, the samples in its input be produced by the echo signal (2). In this situation, due to the coherent accumulation of all chips (elementary phase pulses), forming the echo signal (2), the maximal power of the MF's output reaction, caused by the echo signal (2), is:

$$W_{SMF} = U_{ms}^{2} = \left[\sum_{k=0}^{N-1} \mu(k)\mu^{*}(k)\right]^{2}. \quad (8)$$

Analogously, the maximal power of the MMF's output reaction, caused by the echo signal (2), is:

$$W_{SMMF} = q_{\zeta\xi}^{2}(N_{mp}) = \left|\sum_{k=0}^{M-1} \mu'(k)\xi_{M-1-k}\right|^{2}. \quad (9)$$

Methodology of the Study

From (7), (8) and (3), (9) it follows that

$$q_{MF}^2 = \frac{W_{SMF}}{W_{nMF}} = \frac{U_{mS}^2}{\sigma_n^2 U_{ms}} = \frac{U_{ms}}{\sigma_n^2} = \frac{\sum_{k=0}^{N-1} \mu(k)\mu^*(k)}{\sigma_n^2}, \qquad (11)$$

$$q_{MMF}^2 = \frac{W_{SMMF}}{W_{nMMF}} = \frac{q_{\zeta\xi}^2(N_{mp})}{\sigma_n^2 \sum_{k=0}^{M-1} \xi_k \xi_k^*} = \frac{q_{\zeta\xi}^2(N_{mp})}{E_{MMF}\sigma_n^2}. \qquad (12)$$

In (11) and (12) q_{MF}^2 and q_{MMF}^2 are the SNRs in the outputs of the MF and the MMF respectively, when the noise at the inputs of the MF and the MMF respectively is AGWN. It is accepted the effectiveness of a MMF to be estimated by its coefficient of losses in SNR :

$$\gamma = \frac{q_{MF}^2}{q_{MMF}^2} = \frac{\left[\sum_{k=0}^{N-1} \mu(k)\mu^*(k)\right] E_{MMF}}{q_{\zeta\xi}^2(N_{mp})}. \qquad (14)$$

Methodology of the Study

In pulse mode of radar operation an echo signal passes through the MMF, shown on fig. 1, within $M+N-1$ clock-intervals (clocks). Without loss of generality they can be numbered as $n=0,1,\dots,M+N-2$. This allows the processing of an echo signal by the MMF, shown on fig. 1, to be presented in the following matrix form:

$$\boldsymbol{A_S X} = \boldsymbol{P_{\mu\xi}}. \qquad (22)$$

Methodology of the Study

In (22) $\boldsymbol{A_S}$ is the matrix, describing the movement of the echo signal through the MMF. Due to this reason it consists of the following $N + M - 1$ rows and M columns:

$$\boldsymbol{A_S} = \begin{pmatrix} \mu(0) & 0 & 0 & 0 & 0 & \cdots & 0 \\ \mu(1) & \mu(0) & 0 & 0 & 0 & \vdots & 0 \\ \mu(2) & \mu(1) & \mu(0) & 0 & 0 & & 0 \\ \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots \\ 0 & \mu(N-1) & \cdots & \mu(1) & \mu(0) & \cdots & 0 \\ \cdots & \cdots & & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 & & \mu(N-1) \end{pmatrix} \quad (23)$$

Methodology of the Study

The vector-column $\boldsymbol{X}$ contains the coefficients of amplification of amplifiers on fig. 1, which are the samples of the MMF's transfer function (TF). In its turn, the vector-column $\boldsymbol{P_{\mu\xi}}$ presents the cross-correlation function (CCF) of the echo signal (2) and the MMF's TF :

$$\boldsymbol{X} = \begin{pmatrix} \xi_0 \\ \xi_1 \\ \xi_2 \\ \dots \\ \xi_k \\ \dots \\ \xi_{M-1} \end{pmatrix}, \quad \boldsymbol{P_{\mu\xi}} = \begin{pmatrix} P_{\mu\xi}(-M+1) \\ P_{\mu\xi}(-M+2) \\ P_{\mu\xi}(-M+3) \\ \dots \\ P_{\mu\xi}(k) \\ \dots \\ P_{\mu\xi}(N-1) \end{pmatrix}. \qquad (24)$$

Results of the Study

In the process of the MMFs' synthesis the matrix $\boldsymbol{A_S}$ is known as it is fully defined by the input signal (2) of the MMF. Besides, the vector-column $\boldsymbol{X}$ has to be find, supposing that the vector-column $\boldsymbol{P_{\mu\xi}}$ has the form, providing the maximal possible resolution of echo signals:

$$\boldsymbol{P_{\mu\xi}} = \begin{pmatrix} P_{\mu\xi}(-M+1) \\ 0 \\ \cdots \\ P_{\mu\xi}(N_{mp}) \\ \cdots \\ 0 \\ P_{\mu\xi}(N-1) \end{pmatrix}. \quad (25)$$

Results of the Study

From the above analysis it follows that (22) is equivalent to a system of $N + M - 1$ linear equations with M unknowns. In the most general situation such systems cannot be solved as the quantity $N + M - 1$ of the linear equations ever is greater (except for the trivial case $N = 1$) than the quantity M of the unknowns.

For overcoming the above principle restriction in the classic explorations two methods for synthesis of MMFs for processing echo signals in pulse mode of radar operation have been developed. The both classic methods do not eliminate all the sidelobes of the CCF of the signal (2) and the MMF's TF.

Results of the Study

In this connection, it should be pointed out especially that every combination of mixed echo signals, which times of arrival in the radar receiver differ by more than τ_{ch}, can be resolved correctly if the CCF of the signal (2) and the MMF's TF possesses a zero correlation zone with sufficiently large length. This fact leads to the conclusion that altogether $N-1$ equations in the beginning and the end of the system (23) could be excluded. As a result, the quantity of the equations of the reduced system becomes equivalent to the quantity M of the unknowns and it can be solved ever when the reduced matrix $\boldsymbol{A'}_{\boldsymbol{S}}$ is not singular. The last conclusion will be explained by the following example.

Results of the Study

Example 1. Let the PM signal (2) be $\{\mu(k)\}_{k=0}^{2} = \{\mu(0) = 1, \mu(1) = 1, \mu(2) = -1\}$. (28). Besides, let the parameters of the MMF, which has to synthesized, be $M = 5$, $P_{\mu\xi}\left(N_{mp}\right) = 4, N_{mp} = -1$, (according to (24), here a numeration analogous to the numeration of the ACF's lobes is used). In this situation the matrix (23) is:

$$\boldsymbol{A}_{\boldsymbol{S}} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}. \quad (29)$$

Results of the Study

Besides, $\boldsymbol{X}$ and $\boldsymbol{P}_{\boldsymbol{\mu\eta}}$ are the following vector-columns:

$$\boldsymbol{X} = \begin{pmatrix} \xi_0 \\ \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{pmatrix}, \quad \boldsymbol{P}_{\boldsymbol{\mu\xi}} = \begin{pmatrix} P_{\mu\xi}(-4) \neq 0 \\ P_{\mu\xi}(-3) = 0 \\ P_{\mu\xi}(-2) = 0 \\ P_{\mu\xi}(-1) = 4 \\ P_{\mu\xi}(0) = 0 \\ P_{\mu\xi}(1) = 0 \\ P_{\mu\xi}(2) \neq 0 \end{pmatrix}. \qquad (30)$$

Results of the Study

As $\frac{N-1}{2}=1$ it is convenient the first and the last equations of the system of linear equations, which correspond to the above parameters, to be excluded. As a result, the matrix (29) and the vector-column (30) are reduced to:

$$\boldsymbol{A'_s}=\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}, \boldsymbol{P'_{\mu\xi}}=\begin{pmatrix} P_{\mu\xi}(-3)=0 \\ P_{\mu\xi}(-2)=0 \\ P_{\mu\xi}(-1)=4 \\ P_{\mu\xi}(0)=0 \\ P_{\mu\xi}(1)=0 \end{pmatrix} \quad (31)$$

Results of the Study

Now it is not hard the reduced version

$$\boldsymbol{A'_S X = P'_{\mu\xi}} \qquad (32)$$

of the matrix equation (22) to be solved. The result:

$$\begin{pmatrix} \xi_0 \\ \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{pmatrix} = \boldsymbol{X} = (\boldsymbol{A'_S})^{-1}\boldsymbol{P'_{\mu\xi}} = \begin{pmatrix} 1 \\ -1 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \quad (33)$$

ends the process of MMF's synthesis. Besides, it should be noted that the coefficient of losses in SNR, calculated according to (14), is $\gamma = 1{,}5$.

Conclusion

In the paper a method for synthesis of MMFs, providing zero-correlation zones with arbitrary lengths in the CCF of the radar probe pulse and the MMF’s TF, is developed. As a result, the MMFs, synthesized by the method, could be used as “electronic lenses” for solving complex indeterminate situations during the secondary processing of radar information when the characteristics of group targets have to be determined correctly. Besides, the last possibility allows exploitation in the radar sensor networks many classes of PM signals with complex inner structure, preventing their imitation by cyber-attacks, despite that the autocorrelation properties of these signals may not be optimal.