ON THE EFFICIENCY OF THE EVOLUTIONARY APPROACH TO DYNAMIC OBJECTS IDENTIFICATION IN THE FORM OF DIFFERENTIAL EQUATIONS SYSTEMS

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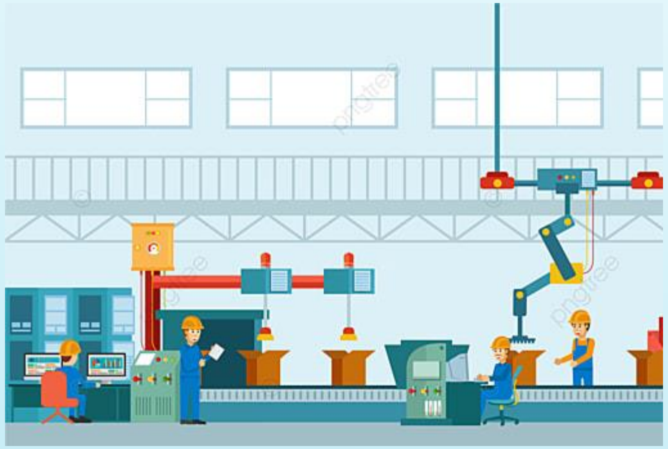
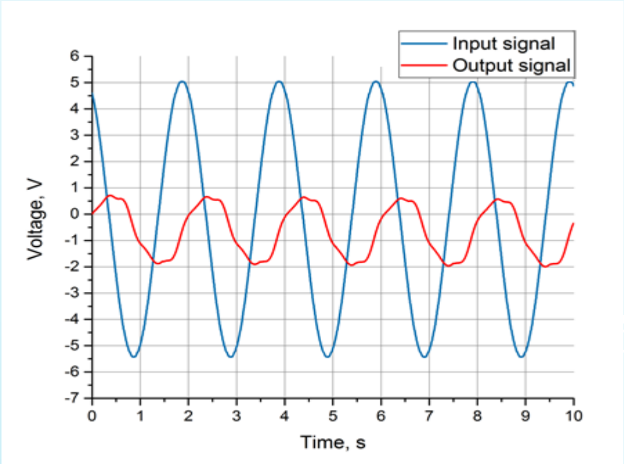
International Conference on Information Technologies (InfoTech), 2024

INTRODUCTION

The development of methods for modeling dynamic objects has long been an important field. It is impossible to conduct research, study the properties of systems, make predictions, and select correct control without an adequate mathematical model.





INTRODUCTION

The process of building a mathematical model of a dynamic system based on measurements of input and output variables characterizing the system’s behavior under study is called **identification**.
One of the most common ways to represent models of dynamic systems is their description with **differential equations**.

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The key advantage of representing models in the form of differential equations and their systems is the ability to obtain the object’s description in a symbolic form that **can be interpreted by a human**.

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The structural-parametric identification of dynamic processes in the form of differential equations systems is examined.
The algorithmic basis of the approach under consideration is evolutionary algorithms.

DESCRIPTION OF THE APPROACH

Problem statement

Solve the inverse problem of mathematical modelling for a process characterized by an arbitrary number of input and output variables, i.e., build a model in the form of a system of differential equations according to the measurements of inputs and outputs:

$$\begin{cases} y_1^{(k)} = f_1(t, x_1, \dots, x_m, y_1, \dots, y_s) \\ y_2^{(k)} = f_2(t, x_1, \dots, x_m, y_1, \dots, y_s) \\ \qquad\qquad \dots \\ y_s^{(k)} = f_s(t, x_1, \dots, x_m, y_1, \dots, y_s) \end{cases}$$

where x_{m_i} are measurements of input variables, y_{s_i} are measurements of output variables, m and s are their numbers, respectively, k is an order of the differential equation.

It is necessary to build a model in the form of a system of differential equations that reflects correctly the relationship between the inputs and outputs of the system; the initial conditions are assumed to be known.

DESCRIPTION OF THE APPROACH

Genetic programming algorithm for the identification of differential equations

It is necessary to specify
the maximum possible order of the differential equation *k*.

Identification problem solving:

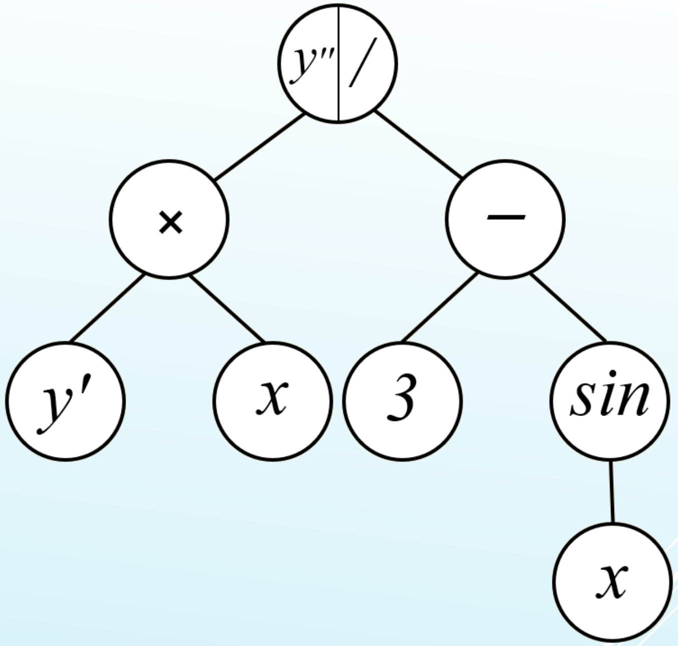
$$\hat{y}^{(k)} = F\left(\hat{y}^{(k-1)}, \dots, \hat{y}', \hat{y}, x\right),$$

where *k* is a differential equation of the order $k<K$ ($k \in N$, *K* is the maximum possible order of the equation, specified by a user).

Functional and terminal sets

Terminal set:
a set of all input *(x)* and output *(y)* variables, a set of constants, derivatives $y', \dots, y^{(k-1)}$.

Functional set:
functions used by the algorithm to form the solution (+, -, /, ×, sin, cos, exp, etc.).



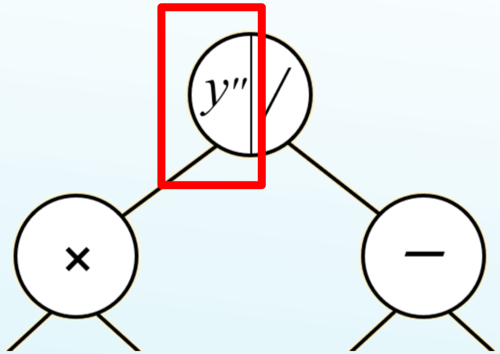
$$y'' = \frac{y'x}{3 - \sin(x)}$$

DESCRIPTION OF THE APPROACH

Modified evolutionary steps

Starting population generation

An order of the differential equation is not higher than the given value of K in the root node.



Fitness function

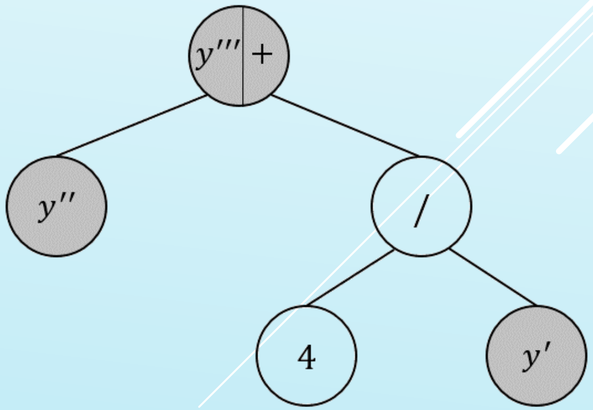
$$fitness = \frac{1}{1 + error},$$

$$error = \frac{1}{n}\sum_{i=1}^{n} \frac{\sqrt{(\hat{y}_i - y_i)^2}}{\max(y_i) - \min(y_i)}.$$

where n is a sample size, $\hat{y}_i$ is a value of the solution at the *i-th* point, y_i is a value from the original sample.

Mutation

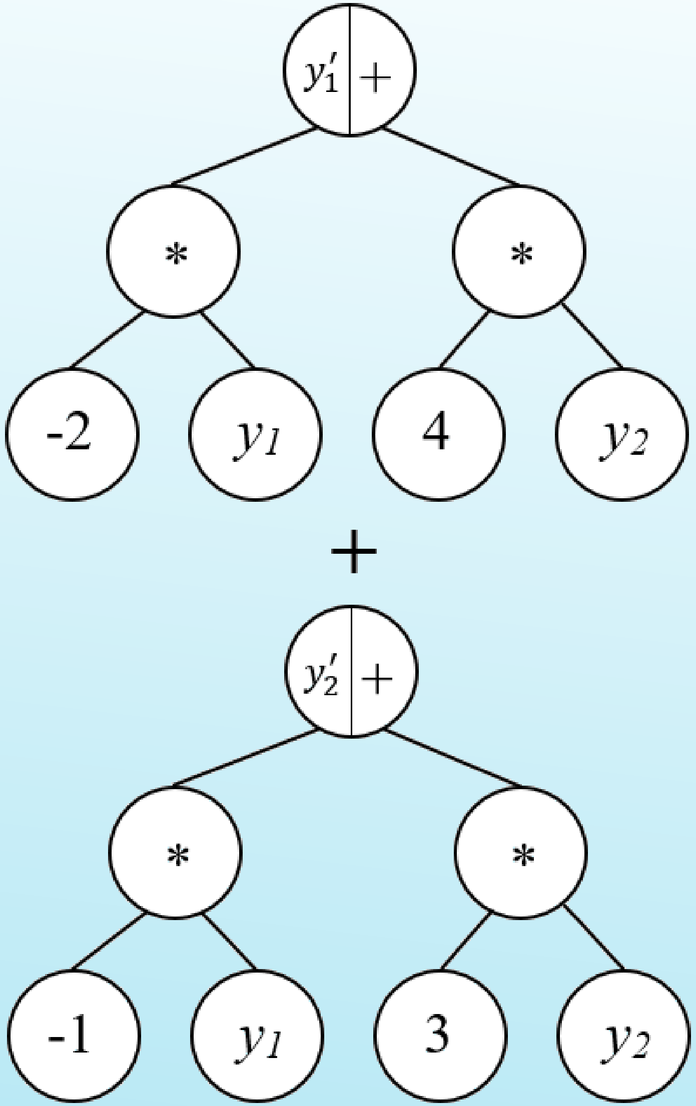
The peculiarity of the mutation in the proposed approach is a possible change in the root node containing the maximum order of the differential equation. In case of a change (mutation) of the maximum order of the differential equation, the order of the derivative must also be changed.



Vertices mutating in changing the root node

DESCRIPTION OF THE APPROACH

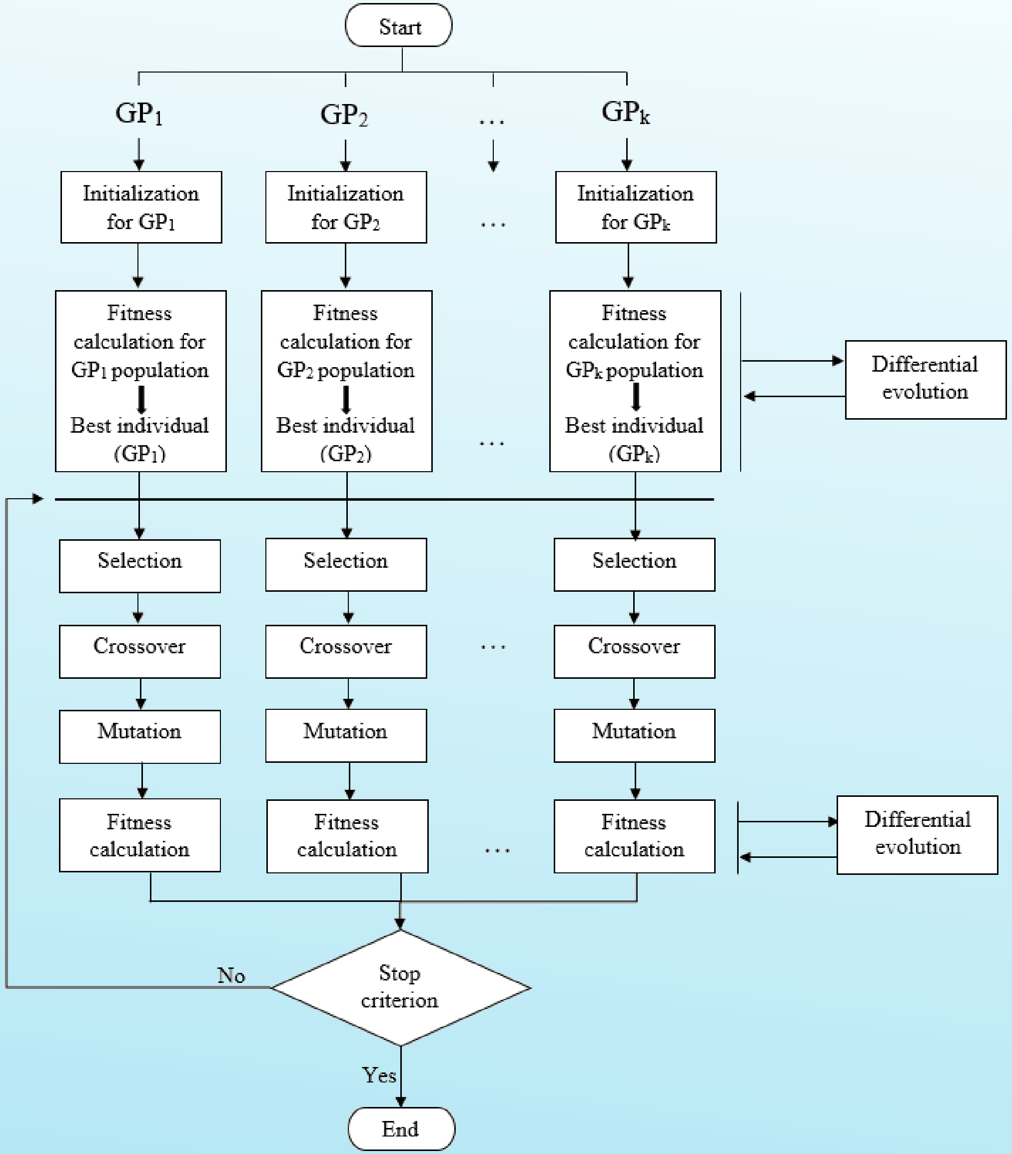
Example of the differential equations system encoded in the form of trees



$$\begin{cases} \dfrac{dy_1}{dt} = -2y_1 + 4y_2 \\ \dfrac{dy_2}{dt} = -y_1 + 3y_2 \end{cases}$$

DESCRIPTION OF THE APPROACH

Identification of differential equations systems



Step 0. Initialization

Step 1. Evolutionary steps

1.1 selection;

1.2 crossover;

1.3 mutation.

Step 2. Fitness calculation

$$fitness = \frac{1}{1 - error},$$

$$error = \frac{\sum_{i=1}^{k} \sum_{j=1}^{n} \left(y_{ij} - \hat{y}_{ij}\right)^2}{kn},$$

where n is a sample size, k is a number of equations, y_{ij} are values from the original sample, $\hat{y}_{ij}$ are values of the model.

Step 3. Checking the stop criterion

DESCRIPTION OF THE APPROACH

Self-configuring of GP and DE

Operator groups

	Type selected operators	*Numerical values elected operators*
GP	Selection (Tournament, rank selection, proportional) Crossover (Standard, single-point, uniform)	Crossover probability Mutation probability
DE	Mutation (Rand1, Rand2, Best1, Best2, Current to Rand, Current to Best)	Differential weight Crossover probability

Algorithms

Population-Level Dynamic Probabilities method

The probability of choosing an operator:

$$par_i = par_{all} + r_i \frac{(100 - n \cdot par_{all})}{scale},$$

where $par_{all} = \frac{20}{n}$, n is a number of operators, $r_i = \frac{success_i^2}{used_i}$, $scale = \sum_{j=1}^{n} r_i$, $used_i$ is a number indicating how many times an operator *i* has been applied, $success_i$ a number of times that the operator *i* succeeded, i.e., the fitness of the offspring was better than the average fitness of the parent individuals.

Success History Adaptation algorithm

The setting of numerical parameters is carried out according to the algorithm Success History Adaptation:

$$new_{val} = \frac{old_{val} + \frac{c_1}{c_2}}{2},$$

where new_{val} is a new parameter value *P*, but old_{val} is a previous parameter value.

$$c_1 = \sum_{i=1}^{n} w_i \cdot (success\_val_i)^2, \qquad c_2 = \sum_{i=1}^{n} w_i \cdot success\_val_i, \qquad w_i = \frac{FitDif_i}{\sum_{j=1}^{n} FitDif_j},$$

where n is a number of successful applications of the custom parameter val, i.e., when the fitness of the offspring is higher than the average fitness of the parent individuals; $success\_ val$ is a value of successfully applied parameter val; *FitDif* is a fitness change for each successful parameter.

TESTING THE APPROACH

Problems represented by differential equations systems of the 1st and 2nd order with different numbers of equations in the system were applied to test the proposed approach.
The dependence of the error of the resulting solutions on the presence of noise in the data and its volume was studied on test problems.

Parameters of algorithm:

- 100 individuals were selected.
- number of generations was 200.
- runs on each function were 30.

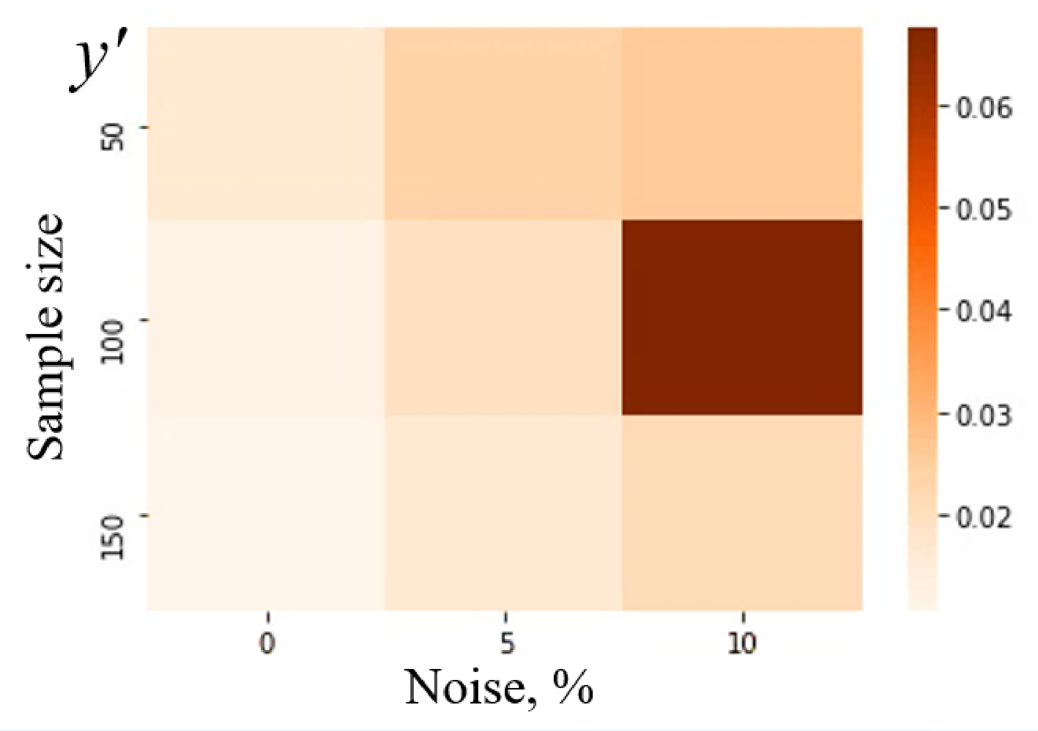
The error is taken as the standard deviation of the output of the differential equation from the original sample.

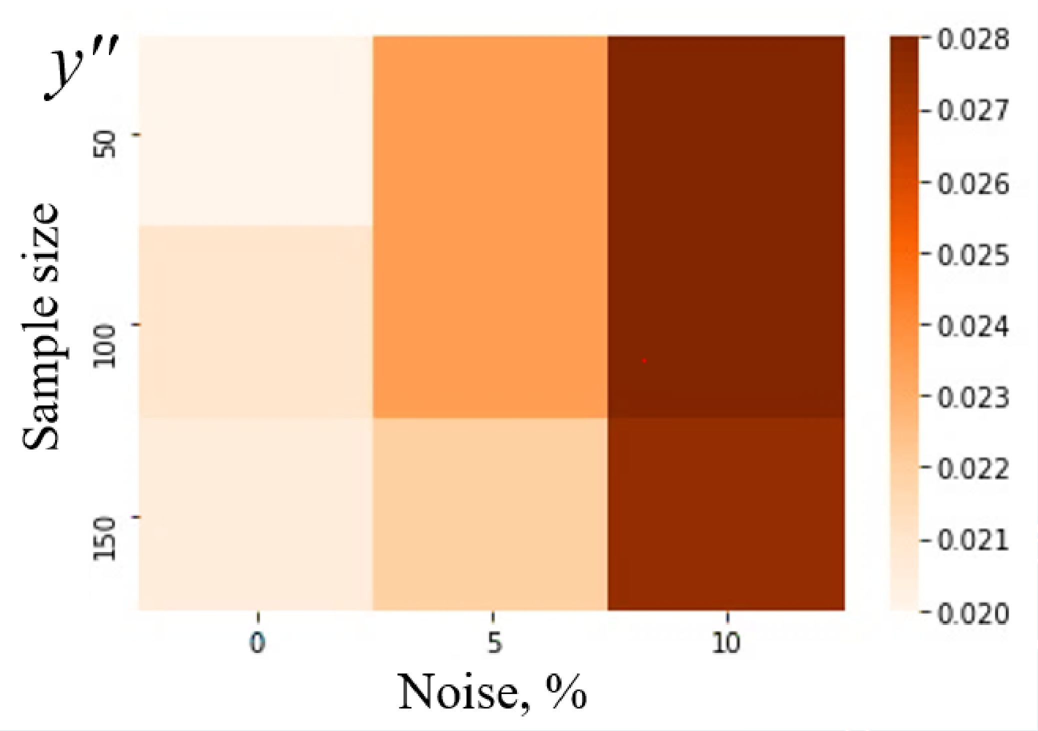
Data:

- Without noise in sample data;
- With noise in sample data (noise equal to 5%, 10%, distributed according to the normal law).

TESTING THE APPROACH

Error of models depending on the sample size and noise level





Figures present results according to error values (averaged over 100 runs) depending on a sample size and the noise presence in the data.

TESTING THE APPROACH

Modeling of longitudinal perturbed motion of an aircraft

It is necessary to obtain a model applying the developed algorithm based on data describing changes in speed v, angle of attack α, angle between the wing chord and the horizon β, and pitch angle over time φ.

The well-known theoretical model is described by the system of equations:

$$\begin{cases} \dfrac{dv}{dt} = -k_{v1}v + k_{v2}\alpha - k_{v3}\beta \\ \dfrac{d\alpha}{dt} = -k_{\alpha1}v - k_{\alpha2}\alpha + \varphi \\ \dfrac{d\beta}{dt} = \varphi \\ \dfrac{d\varphi}{dt} = -k_{\varphi1}v - k_{\varphi2}\alpha - k_{\varphi3}\varphi \end{cases}.$$

TESTING THE APPROACH

Modeling of longitudinal perturbed motion of an aircraft

The models obtained for data with noise of 5% and 10% have an accuracy of 0.95 (1) and 0.945 (2), respectively:

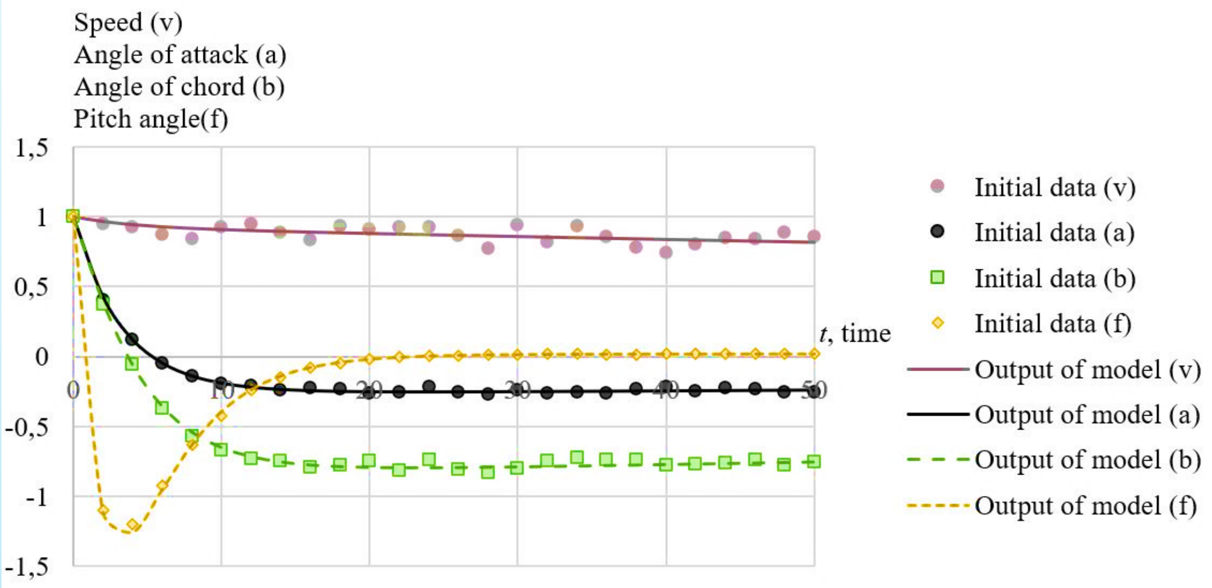
$$\begin{cases} \dfrac{dv}{dt} = 0{,}1 - 0{,}111\beta \\ \dfrac{d\alpha}{dt} = -\dfrac{\varphi}{0{,}207} \\ \dfrac{d\beta}{dt} = -2{,}306\beta - 1{,}684 \\ \dfrac{d\varphi}{dt} = -4{,}165\varphi - 1{,}648\varphi^2 + \dfrac{\varphi}{v} \end{cases} \quad (1),$$

$$\begin{cases} \dfrac{dv}{dt} = -0{,}188\beta - 0{,}154 \\ \dfrac{d\alpha}{dt} = -2{,}544\beta - 1{,}832 + \varphi \\ \dfrac{d\beta}{dt} = -3{,}609 - 7{,}321\beta - 7{,}321\alpha \\ \dfrac{d\varphi}{dt} = -4{,}115\varphi - 0{,}032 \end{cases} \quad (2).$$

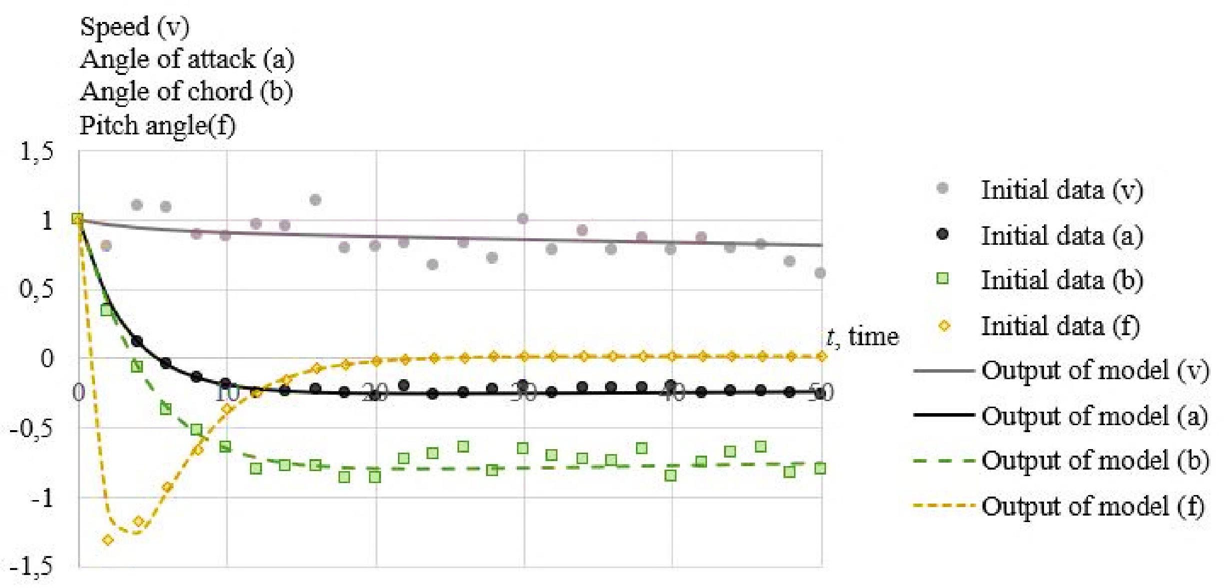
TESTING THE APPROACH

Modeling of longitudinal perturbed motion of an aircraft

Correspondence of the original sample to the output of the resulting model



sample size is 25, noise is 5%



sample size is 25, noise is 10%

TESTING THE APPROACH

Modeling the viscous fluid movement when heated it from below

Consider peculiarities for the process of a viscous liquid movement when heated it from below. There is a temperature difference in the upper and lower layers when a liquid is heated from below. It leads to the fact that the warm liquid floats up, and the cold liquid, on the contrary, sinks. According to the physics of the process, horizontal cylindrical shafts can appear in a liquid, rotating towards each other. When heating is increasing, each shaft breaks up into two smaller shafts, etc., so that in the limit chaos (turbulence) can arise.

A nonlinear system of differential equations called the Lorentz system describes this process as follows:

$$\begin{cases} \dfrac{dx}{dt} = \sigma(y - x) \\ \dfrac{dy}{dt} = x(r - z) - y, \\ \dfrac{dz}{dt} = xy - bz \end{cases}$$

where t is time; x is rotation speed of the shafts, y is temperature difference between the layers, z characterizes the deviation of the vertical temperature profile from a linear dependence; r, σ, b are parameters proportional to the Prandtl number, Rayleigh number and a certain coefficient reflecting the geometry of the region.

TESTING THE APPROACH

Modeling the viscous fluid movement when heated it from below

For data consisting of 50 points without noise, a model identical to the known true one was obtained. The following models with the smallest error were obtained for the remaining data sets:

$$\begin{cases} \dfrac{dx}{dt} = 10{,}002(y - x) \\ \dfrac{dy}{dt} = x(9{,}999 - z) - y, \\ \dfrac{dz}{dt} = xy - 2{,}671z \end{cases}$$

sample size is 50, noise is 5%

$$\begin{cases} \dfrac{dx}{dt} = z - \dfrac{x^2}{0{,}937} + 16{,}262 \\ \dfrac{dy}{dt} = \dfrac{x(8{,}761 - z) + \dfrac{x}{y}}{1{,}552}, \\ \dfrac{dz}{dt} = \dfrac{y(y - 0{,}034 - x)}{0{,}418} \end{cases}$$

sample size is 25, noise is 0

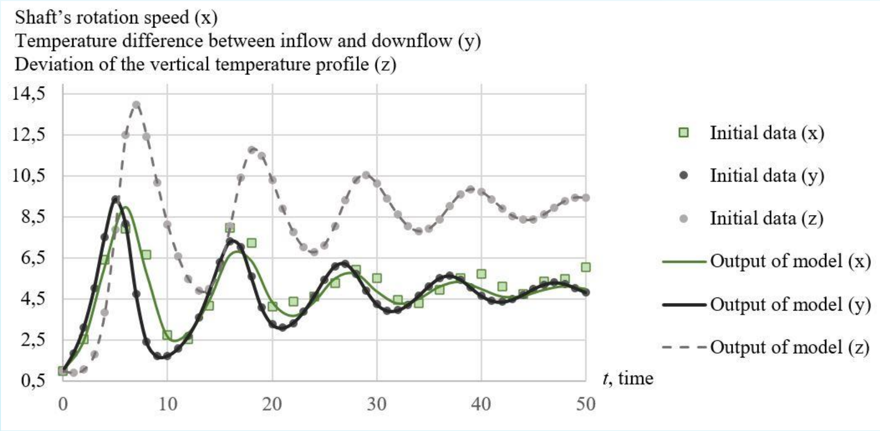
$$\begin{cases} \dfrac{dx}{dt} = 13{,}893 - z - y \\ \dfrac{dy}{dt} = y(1{,}897x - z) \\ \dfrac{dz}{dt} = z - x(z - 1{,}467y) \end{cases} .$$

sample size is 25, noise is 5%

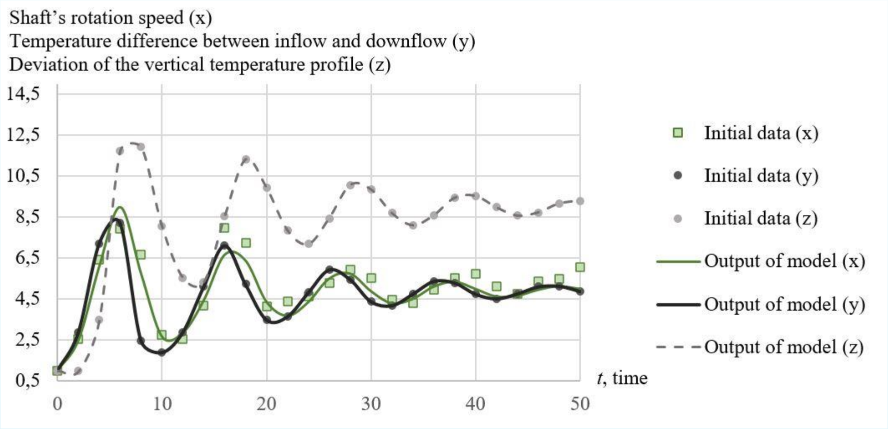
TESTING THE APPROACH

Modeling the viscous fluid movement when heated it from below

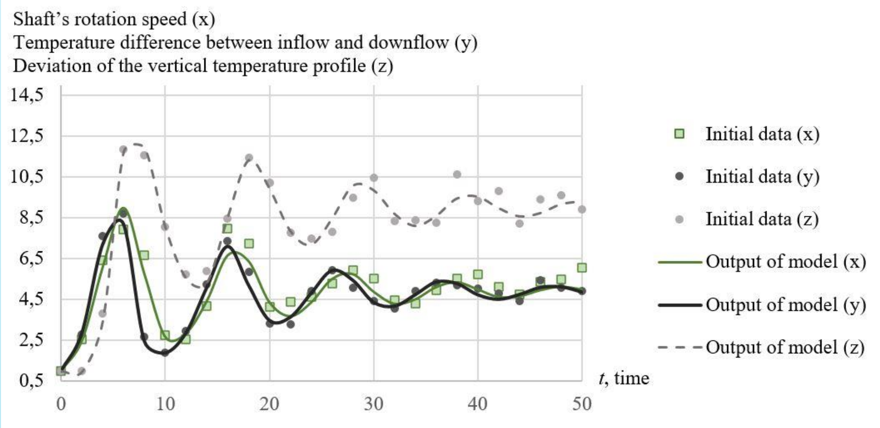
Correspondence of the original sample to the output of the resulting model



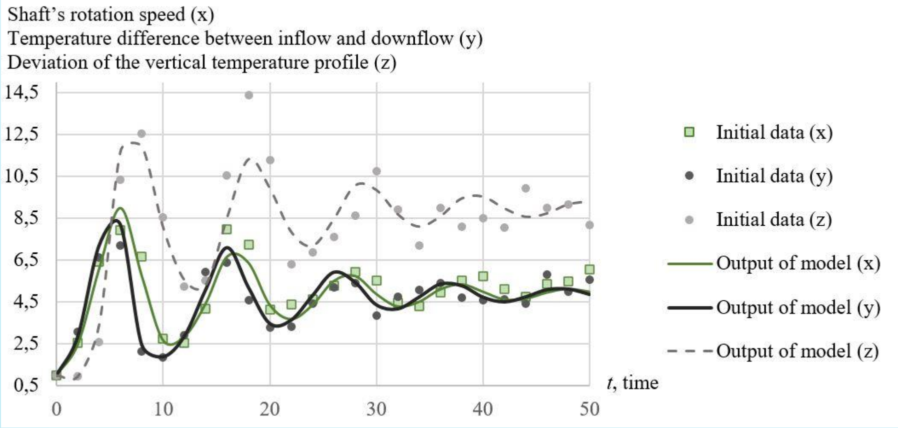
sample size is 50, noise is 5%



sample size is 25, noise is 0



sample size is 25, noise is 5%



sample size is 25, noise is 10%

CONCLUSION

- The work examines the efficiency of the evolutionary approach application to the structural-parametric identification of dynamic objects in the form of systems of differential equations.
- Obviously, the approach has its drawbacks. Thus, the desire to minimize calculation errors can lead to the application in the proposed models of a great number of free parameters. Their physical meaning is often difficult to interpret.
- The key peculiarity of this approach is the ability to evaluate the correctness of the model due to its symbolic representation.
- The work demonstrates a solution to the applied problem and proves that it is possible to obtain a structure corresponding to the initial truth. However, a degree of accuracy is high even at obtaining approximating equations.

THANKS FOR ATTENTION!